A Universal Turing Machine
Does Not Predict All Physical Events.

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Abstract: Paul Davies is the author of "The Mind of God: The Scientific Basis for a Rational World." Davies is a "Templeton Prize" winner. The Davies hypothesis is *Applications of physical laws yield universal Turing machine computable physical events and, hence, the universe lends itself to simulation via such a machine.* That is, such a machine predicts all physical behavior as would a Theory of Everything. It is shown that this hypothesis requires additional restrictions in order to be meaningful. Then brain produced objective mental images are used to refute Davies' universal Turing machine hypothesis. It is also shown that, within the external universe, described physical events as produced by physical laws are not $\mathcal{U}$-computed. This also refutes the Davies' Turing machine hypothesis.

Can the modern formal set-theory $\mathcal{S} = \mathrm{ZFC}$ (the ZF axioms and the C the Axiom of Choice) be informally shown to be consistent? There does exist an informal **conceptual** model $\mathcal{M}_{\mathcal{S}}$ that uses the concepts of *platforms where formation takes place, before or after a platform* and *formed sets occur at each platform.* The intuitive concepts for the words "before" and "after" are the concepts employed. As used here, an object can neither be before nor after itself. General rules that allow one to construct sets via what "came before" and what "has come after" are allowed by Shoenfield (1977, p. 323). Using these concepts, these axioms of ZF set-theory, as Shoenfield presents them, appear to be consistent. In what follows, the more easily imagined white road and panes method is used as a model for the "completed set" of natural numbers (Herrmann, (2013), and Appendix). Throughout all that follows, mental images that can be reproduced by others are necessarily considered as directly related to physical events within the human brain.

Since models were first introduced within mathematical logic, certain basic aspects of the informal natural numbers have been allowed. If this were not the case, such results as Gödel's Incompleteness Theorem can not be convincingly established. When this is done, simple aspects of set-theory can also be used. The major one is the idea that we can imagine or even write down two symbols { and } and place between them other symbols. For example, $\{\mathrm{g}, \mathrm{h}, \mathrm{very}\} = \mathrm{X}$. Then intuitively the symbols $\mathrm{g} \in \mathrm{X}$ mean

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that the symbol g is within the { and } either first or last or between two "commas." The g is called an "element" or "member of." Of course, the symbols can be but names for other objects, physical or imaginary. When something is not within { and }, say the symbol M, then this is expressed as $\mathrm{M} \notin \mathrm{X}$. The X is called a "constructed set."

If the members of the set X are members of set Z, then this is written as $\mathrm{X} \subset \mathrm{Z}$. Of course, these concepts have been used for over a hundred years. Notice that $\subset$ has a few simple properties relative to objects placed between { and }. If $\mathrm{X} \subset \mathrm{Y}$ and $\mathrm{Y} \subset \mathrm{Z}$, then $\mathrm{X} \subset \mathrm{Z}$. Further, $\mathrm{X} \subset \mathrm{X}$. There is also the idea of the "union" operation between such constructed sets. If X and Y are constructed sets, then $\mathrm{X} \cup \mathrm{Y}$ is the constructed set you get by placing between { and } the members of X as well as the members of Y. Thus $\{\mathrm{g}, \mathrm{h}, \mathrm{very}\} \cup \{2, 3, 4\} = \{2, 4, \mathrm{g}, \mathrm{very}, \mathrm{h}, 3\}$. Notice that the idea of "placing things between { and }" does not require them to be "ordered" in any manner. That is, it is not required that an object be placed within { and } "prior" to another. If we can identify members of a set individually, then we only note that they are "members of the set." There is a symbol that indicates that "nothing has been placed between { and }". This is called the "empty set" and is denoted by $\emptyset$. In this case, the symbols { and } are not used.

"Equal" constructed sets, denoted by =, have the exact same members. Thus $\{\mathrm{g}, \mathrm{h}, \mathrm{very}\}$ and $\{\mathrm{very}, \mathrm{g}, \mathrm{h}\}$ are equal. That is, $\{\mathrm{g}, \mathrm{h}, \mathrm{very}\} = \{\mathrm{very}, \mathrm{g}, \mathrm{h}\}$. So, constructed sets X and Y are equal if and only if $\mathrm{X} \subset \mathrm{Y}$ and $\mathrm{Y} \subset \mathrm{X}$. Or $\mathrm{x} \in \mathrm{X}$ if and only if $\mathrm{x} \in \mathrm{Y}$. If there are two such empty sets $\emptyset_1$, $\emptyset_2$, then since there are no members in either, $\emptyset_1 \subset \emptyset_2$ and $\emptyset_2 \subset \emptyset_1$ imply that they are equal and one symbol is sufficient. Notice that for the same reason, for any set X, $\emptyset \subset \mathrm{X}$.

A specific collection of panes with specific objects attached, as mentally viewed via the discussed procedures in Herrmann, (2013) and in the Appendix, is denoted by $\mathcal{Q}$. For ZF, $\mathcal{Q}$ is used as a model for the Infinity Axiom, a completed infinity.

For the ZFC set-theory axioms, Shoenfield does not describe in a specific way a special object that satisfies the Infinity Axiom. He simply states, after constructing a set via a direct rule, "Suppose that x_n is formed at S_n, then there is a stage after all the S_n. At this stage we can form the set x whose members are $x_0, x_1, \ldots$." (Shoenfield (1977), p. 326.) This is not very convincing. The natural number notation can be replaced with a "tick" notation. But this basic object still relies upon the intuitive notation for the counting numbers (i.e. an intuitive aspect of the natural numbers). The "counting" notion as a prior concept is eliminated for the imagined objects described in the Appendix and the needed set is convincingly obtained via the $\mathcal{Q}$ view. Once $\mathcal{Q}$ is obtained, one is allowed to use certain intuitive properties of the intuitive natural numbers.

An important aspect of the construction of $\mathcal{Q}$ is that while constructing each member, I retain in some form a "mental intention" as to why I am constructing them. This mental intention is mentally fulfilled without any additional instruction being given.

Recall that, in ZF, given two sets, X and Y, then $\mathrm{z} \in \mathrm{Z} = \mathrm{X} \cup \mathrm{Y}$ if and only if $\mathrm{z} \in \mathrm{X}$ or $\mathrm{z} \in \mathrm{Y}$. Since set $\emptyset$ exists within ZF set-theory, then the Infinity Axiom can be stated as follows:

> **Let $\mathbf{x}' = \mathbf{x} \cup \{\mathbf{x}\}$. There exists a set X such that $\emptyset \in \mathbf{X}$ and, for each x, $\mathbf{x} \in \mathbf{X}$ if and only if $\mathbf{x}' \in \mathbf{X}$. This set is unique and the members of X have a unique form. This unique set is formally denoted by ω. Below is described the $\mathcal{Q}$ class that satisfies this axiom. It is shown that such a constructed class, at the least, exists mentally.**

A "constructed number" is a special and direct why to describe objects that are on or attached to the panes. You view the pane-by-pane construction in the direction of the right vanishing point. You begin the step-by-step construction process of non-equal sets as follows: On the pane at the "start" line, you have attached the $\emptyset$ or, at least its symbol. Then on the successive panes, you follow the rule: **take the previous object on the previous pane and put the "previous object $\cup$ {previous object}" on it.** So, you put on the next pane $\emptyset \cup \{\emptyset\} = \{\emptyset\}$. Then on the next pane you put $\{\emptyset\} \cup \{\{\emptyset\}\} = \{\emptyset, \{\emptyset\}\}$. Then observing the pattern, on the next pane put $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$. This simple process only involves the previous pane. Thus, mentally, this can be continued. Each construction is a "constructed number."

Imagining and viewing the white road with these constructed numbers attached to the panes is the "class" of constructed numbers $\mathcal{Q}$. It is precisely the constructed numbers that obviously model the Infinity Axiom for ZFC. By defining relations on the members of $\mathcal{Q}$, the $\mathcal{Q}$ view becomes a model for the completed class of (natural) numbers. This contradicts those that claim they are not imaginable.

Notice that the constructed numbers satisfy the formal statement

$$\mathrm{Q}^* = \exists \mathrm{y}((\emptyset \in \mathrm{y}) \wedge (\forall \mathrm{x}(\mathrm{x} \in \mathrm{y} \leftrightarrow \mathrm{x}' \in \mathrm{y})))$$

We learn early in life the symbols and names given to objects when one repeats the process of combining a single object with a finite collection of others. It is a step-by-step rule starting with an empty collection. We learn the concept of counting and "adding one more" object to a collection of objects. For a constructed number x on a pane, what is placed on the next pane, $x \cup \{x\}$, has an abbreviated name x'. We can use the counting numbers as names for the items on a pane via the "next prime" rule. The intuitive counting numbers are used to indicate the "number" of successive applications of the "next prime" rule. In terms of the successive application of the prime notation, we have, where $\equiv$ means an "equivalent name," that $\emptyset' \equiv \emptyset^{(1)}$, $(\emptyset')' \equiv \emptyset^{(2)}$ and the like (Hamilton (1978, p. 126)). Notice that $\emptyset^{(0)} = \emptyset$ the symbol with no prime. This allows us to use a symbol to indicate how many successive applications of the prime operation it takes to produce the pane objects. Take 0 from the intuitive set of natural numbers $\mathbb{N}$. Let 0 denote $\emptyset$. Then there are the corresponding $0^{(0)}, 0^{(1)}, 0^{(2)}, 0^{(3)}, 0^{(4)}, \ldots, 0^{(10000)}, \ldots$ notations. There is one symbol being used here that must be interpreted relative to the model being discussed. The 0 is a symbol used for a constant in **S**. And, in this class, it is used for the empty set. The context should indicate which is which if necessary.

There is a new view that corresponds to $\mathcal{Q}$. It is the white road and, in place of the panes, prime notations as here abbreviated or considered as informally modeled by $0^{(n)}$ are placed on the road perpendicular to it. They are placed one after the other and they are directed towards the vanishing point. As we view from the background, these symbols, sitting one behind the other on the white road, appear to get smaller and smaller as they are formed in the direction of the vanishing point. But, just as done for the panes, we know that no matter where we concentrate our view on the road there is such a symbol produced by the "next prime" rule. Call this view $\mathcal{Q}'$.

There is this relation between members of $\mathcal{Q}$ and $\mathcal{Q}'$. Each member of $\mathcal{Q}$ corresponds to one and only one member of $\mathcal{Q}'$. Each $x \in \mathcal{Q}$, other than $\emptyset$, is constructed by successive applications of $'$. Thus, correspond each member of $\mathcal{Q}'$ to these symbolic forms and conversely. Indeed, as described in the Appendix, there are other types of classes that are $\mathcal{Q}$ related in this manner. Notice that we can put an "order" on these symbols. For $\mathcal{Q}$, there is an order placed on the constructed numbers. Take any constructed number x, then another constructed number y is either the same as x or immediately before x or immediately after x. This corresponds to the $\in$ and $\subset$ simple orders for the constructed numbers. Now consider the informal nature numbers with the order $\leq$ defined. This induces an order on $\mathcal{Q}'$, where $\emptyset^{(n)} \leq \emptyset^{(m)}$ if and only if $n \leq m$. This order preserves, with 0 substituted for $\emptyset$, a simple order for members of $\mathcal{Q}'$. It should be clear that the two views are slowly taking on the same properties as the intuitive natural numbers via corresponding properties for $\mathcal{Q}'$.

From these consistent approaches used to develop these classes, each member of $\mathcal{Q}$ corresponds to a member of $\mathcal{Q}'$ and conversely. This correspondence preserves a simple order. Thus, now consider $\mathcal{Q}'$ as corresponding to a completed class of natural numbers. For ZFC, $\mathcal{Q}'$ corresponds to formal ω. You can view $\mathcal{Q}'$ in two ways, the repeated prime form or the representative informal natural number form.

To apply the Shoenfield notion for set formation, another white road with panes is necessary. Both white roads are necessary in order to properly model ZF. It can be included in the mental view, but one of the symbolic names for $\mathcal{Q}'$ needs to be used. The reason for this is that on the "first start" pane the entire $\mathcal{Q}'$ needs to be attached. Shoenfield gives the general statement that "sets are formed from the sets formed before a specific pane" and this formation concept does not, in general, include a fixed rule for such formation such as the rule used to obtain the members of $\mathcal{Q}$. This more general approach is necessary for the other panes on this "new" white road. It is this general approach, with the additional white road and the $\mathcal{Q}$, that is used to "model" the ZF axioms, where, for the constructions, specific ZF axiom inspired rules are used. Since it has been shown that the Axiom of Choice is independent from the other ZF axioms, then the appropriate "choice" sets can simply be added to the mix.

In Mendelson (1987, p. 117) is listed a formal set of axioms **S**. Any axiom system formally equivalent to this is said to yield formal Peano-Arithmetic, **PA**. Theorems such as the Gödel's theorems, as normally presented (Mendelson, (1987)), are metatheorems that use properties of $\mathbb{N}$ to establish. The present goal is to show that $\mathcal{Q}'$ can have properties defined so that $\mathcal{Q}'$ models **S**.

For a model for **S**, the successor operation is but the $'$ operation defined for the members of $\mathcal{Q}$ that corresponds to the appropriate member of $\mathcal{Q}'$. The addition and multiplication concepts (binary operations) are denoted in **S** by f_2 and f_3.

By using allowed properties for the informal natural numbers, $\mathcal{Q}'$ also models the f_2 and f_3 operations. As done by Mendelson (1987, p. 124, Proposition 3.6), and Hamilton (1878, p. 126) define addition $\mathrm{f}_2 = \oplus$ on $\mathcal{Q}'$ as follows: For the intuitive $\mathbb{N}$ notation, consider two constructed numbers $\mathrm{x} = 0^{(\mathrm{n})}$ and $\mathrm{y} = 0^{(\mathrm{m})}$. Now define $\mathrm{f}_2(\mathrm{x}, \mathrm{y}) = \mathrm{x} \oplus \mathrm{y} = 0^{(\mathrm{n}+\mathrm{m})}$. Then define $\mathrm{f}_3(\mathrm{x}, \mathrm{y}) = \mathrm{x} \odot \mathrm{y} = 0^{(\mathrm{n}\times\mathrm{m})}$. The $+$ and $\times$ defined on $\mathbb{N}$ are but the notion as to how to present successive application of the "addition" of 1 to a member of $\mathbb{N}$. For constructed numbers $\mathrm{x} = 0^{(\mathrm{n})}$ and $y = 0^{(\mathrm{m})}$, these two rules yield constructed numbers. From these definitions, the axioms of **S** are modeled by relating the constructed number forms, in this manner, via the original notion of the priming operation. (There are more formal ZF ways to obtain the "sum" and "product" operations, but the results correspond exactly to this informal approach.)

As an example, consider the Induction Axiom, axiom S(9). This is formally stated as

$$\mathcal{A}(0) \to ((\forall x(\mathcal{A}(x) \to \mathcal{A}(x'))) \to \forall x(\mathcal{A}(x))),$$

where x is a free variable in formal $\mathcal{A}(x)$.

For free variable x, let $\mathcal{A}(x)$ be any formal first-order statement defined in terms of the defined $\mathcal{Q}'$ operations. (Such expressions are called "well-formed-formula" (wf).) What it means to say that $\mathcal{A}(x)$ "holds" is that when the other free vairables are considered as arbitrary members of $\mathcal{Q}'$ then, when translated into a statement A(x) about $\mathcal{Q}'$, A(x) is true. The axiom states to first show that $\mathcal{A}(0)$ holds. Then assume that for a member x of $\mathcal{Q}'$, $\mathcal{A}(x)$ holds. From these two known statements, establish that $\mathcal{A}(x')$ holds. Then one can state, in general, that $\mathcal{A}(x)$ holds for an arbitary constructed natural number. (However, for **S**, $\mathcal{A}(x)$ is restricted to specific sets of constructed numbers relative to the model.)

To estabish that this axiom is satisfied by members of $\mathcal{Q}'$, what are the objects being discussed? They are objects in direct correpondence to those in $\mathcal{Q}'$. So, each time we assume or establish that $\mathcal{A}(x)$ holds for x, then this x is a member of $\mathcal{Q}'$. Assuming that $\mathcal{A}(x)$ holds means that this statement corresponds to a member x in $\mathcal{Q}$. From these two statements, it is shown that $\mathcal{A}(x')$ holds. This corresponds directly to x$' = x \cup \{x\}$ in $\mathcal{Q}$ and a member with one additional prime in $\mathcal{Q}'$. But, the class $\mathcal{Q}$ is the only original object that exists as a class and this is its characterizing property. Hence, the same holds for $\mathcal{Q}'$. Thus, A(x), holds for each x $\in \mathcal{Q}'$ and, therefore $\mathcal{A}(x)$. Thus, the Modus Ponens rule for deduction can be applied. But, the generalization must be restricted to appropriate members of $\mathcal{Q}'$ when the term "model" is employed.

Due to how the operations have been defined and this established induction axiom, finite subsets of $\mathcal{Q}'$ form a model for the axiom system **S**. The members of $\mathcal{Q}$ and the corresponding members of $\mathcal{Q}'$ are both called constructed numbers. Using a class, such as $\mathcal{Q}'$ or the corresponding $\mathcal{Q}$, as a model for such axioms is a valid modeling procedure (Jech, (1971)).

The induction axiom is a type of step-by-step induction. Of course, this induction axiom needs to be accepted. It is accepted by mathematicians including the Intuitionists and Constructionists.

> "Thus we can argue as follows: Let $P(x)$ be a property of natural numbers such that $P(1)$ holds [in our case, $P(0)$] and $P(n)$ implies $P(n+1)$. Then the Intuitionist observes that generating k from 1 and passing over to k by the generating process, the property P is preserved at each step and hence holds for k." (Wilder (1967, p. 249))

As mentioned, due to the finite nature of such an induction statement, and since $\mathcal{Q}' \notin \mathcal{Q}'$, the quantifier must be restricted. Hence, what has been shown is that the property holds for an arbitrary finite set of constructed numbers of the form $\{0, 1, 2, 3, 4, \ldots, \mathrm{k}\}$ and not for members of an infinite set of such numbers. That is, you can substitute for k any constructed number. These sets do not contradict the **S** axioms or the finite requirements for writing an **S** wf and the finite number of steps in a computation or deduction.

> A major result used in this article is that **for S, an "informal number-theoretic relation" K is "expressible" if and only if** $\mathrm{C_K}$ **is "representable." (Mendelson (1987, p. 132, Prop. 3.12).** Turing computable functions are identical with the partial recursive functions. Further, for a total number-theoretic function, if it is recursive, then it is partial recursive (Mendelson, (1987, p. 243, Exercise 5.15)). For any total number-theoretic function, if it is recursive, then it is $\mathcal{U}$-computable. And if it is $\mathcal{U}$-computable, then it is recursive (Mendelson (1987, p. 249, Corollary 5.9)).
>
> Since there are number-theoretic functions that are not $\mathcal{U}$-computable, relative to the Turing machine processes and, for such functions, Kleene states, "To improve the procedure or machine must take ingenuity, something that cannot be built into the machine" (Kleene, (1967, p. 246)).

For any formal first-order axiom system K, let $\mathcal{T}(\mathbf{K})$ be the formal first-order theory generated from K. Each quantified wf $\mathcal{B}$ is equivalent to a well-formed-formula (wf) with all the quantifiers on the left of a quantifier free wf $\mathcal{A}$. Each quantifier free $\mathcal{A} \in \mathcal{T}(\mathbf{S})$ corresponds to a quantifier free $\mathcal{A}^*$, in ZFC, where ordinal arithmetic and properties correspond to the arithmetic and properties expressed by members of $\mathcal{T}(\mathbf{S})$. In particular, each axiom of **S** and each member of $\mathcal{T}(\mathbf{S})$ corresponds to a wf in ZFC. Further, each such $\mathcal{A}^*$ corresponds to a special form $\mathcal{A}\#$ in ZFC, where for the free variables $x_1, \ldots, x_n$ in $\mathcal{A}^*$, $\mathcal{A}\#(x_1, \ldots, x_n) : ((x_1 \in \omega) \wedge (x_2 \in \omega) \wedge \cdots \wedge (x_n \in \omega)) \to \mathcal{A}^*$. Then for the quantifiers for $\mathcal{B}$, if any, the same quantifiers yield transformed $\mathcal{B}\#$. Let ZFC# be the ZFC formal language restricted to these forms. Let **S#** be the transformed axioms **S**. When proofs are restricted to the two formal languages, $\vdash_{\mathbf{S}} \mathcal{C}$ if and only if $\vdash_{\mathbf{S}\#} \mathcal{C}\#$ (Mendelson, (1987, p . 208-209) stated for ZFC). Further, under these restrictions, $\mathcal{C}$ is satisfied (for a structure) for **S** if and only $\mathcal{C}\#$ is satisfied (for a structure) for **S#**.

In order to determine whether a number-theoretic expression is true or false relative to **S**, the expressions can be re-expressed in terms of members of ZFC#. Each wf of $\mathcal{T}(\mathbf{S}\#) \subset ZFC\#$ has a coded Gödel number. To relate each wf in $\mathcal{T}(\mathbf{S})$ with the corresponding one in $\mathcal{T}(\mathbf{S}\#)$, define the number-theoretic function h(x) as follows: If x is a Gödel number of the wf $\mathcal{C}$ in $\mathcal{T}(\mathbf{S})$, then h(x) is the Gödel for $\mathcal{C}\# \in \mathcal{T}(\mathbf{S}\#)$. Or if x is not the Gödel number of the wf in $\mathcal{T}(\mathbf{S})$, then h(x) = 0. Since to obtained a member of $\mathcal{T}(\mathbf{S}\#)$ from **S#**, the exact same first-order logic can be employed to obtain a corresponding member of $\mathcal{T}(\mathbf{S})$ from **S**, then h can be considered as one-to-one. This function is recursive and, hence, $\mathcal{U}$-computable.

When one has proper axioms, and one or more are employed, a proof would not be very useful if the variables, terms and constants did not refer to objects that satisfied the axioms. This is why when you state that $\mathcal{A}(x,y)$ is formally proved using **S** axioms, the **S** is included in the symbol $\vdash_{\mathbf{S}} \mathcal{A}(x,y)$. However, this is a one-way procedure for the unrestricted use of quantifiers. Formal expression that are satisfied for members one domain, need not be satisfied when interpreted in a domain for another model for **S**. Indeed, there are "nonstandard models" for **S** for which this is the case. In the $\mathcal{Q}'$ domain form for the constructed numbers, the statement that there does not exist a constructed number y such that for each specific constructed number n, n $\leq$ y holds. Then generalization holds for this statement if it is restricted to the constructed numbers.

There is a predicted domain ${}^*\mathcal{Q}'$ and operations on it that model **S**. This domain contains a copy of $\mathcal{Q}'$. Generalization, $\forall$, holds for members of the domain ${}^*\mathcal{Q}'$, but it does not hold when applied to members of the corresponding domain $\mathcal{Q}'$. There is a y in ${}^*\mathcal{Q}'$ that is greater than each constructed number in $\mathcal{Q}'$. **Thus, when generalization is applied to a statement that holds for an arbitrary member of a domain, it can only be stated that the statement holds for the specific domain being used for the model.** This should be understood about quantifiers but this fact is hardly ever mentioned.

Previously, the phrase "number-theoretic relation" is used. These are relations expressed in terms of the intuitive natural numbers. "Number-theoretic functions and relations are intuitive and are not bound up with any formal system." (Mendelson, (1987, p. 130)). **It is almost never stated that one uses their knowledge of the intuitive natural numbers and, when an expression is written in terms of the allowed operations, it is human mental processes that determine whether the statement is a fact; that it is true.** Then, for systems such as **S**, it is restricted to the defined operations. These operations do correspond to operations definable within the intuitive natural numbers.

The informal natural numbers correspond to $0^{(\mathrm{x})} \equiv \emptyset'^{\cdots\prime}$ and correspond to objects that satisfy the **S** axioms. For a pane, with the natural number name x, the symbol $\overline{x}$ is often used to denote the representation of x via the successor operation in $\mathcal{T}(\mathbf{S})$ with constant 0. The major fact is that if one properly states a number-theoretic relation R informally using the informal operations, then there is a corresponding $\mathcal{T}(\mathbf{S})$ formal statement R. As stated, when informal natural numbers are substituted into R, then, due to the language being used, it must informally be either a true or false statement about the R. (This is also stated by the expression "it satisfies or does not satisfy R.")

The symbols used for the statement that "R is formally proved in **S**" are $\vdash_{\mathbf{S}} R$.

For R to be expressible in **S**, the following must hold. If selections for the informal natural numbers are substituted into R and R is the true, then it must be that $\vdash_{\mathbf{S}} R$ for the corresponding **S** objects. For the case that R is false, it must be the case that $\vdash_{\mathbf{S}} \neg R$. Notice that R can be considered as determining a total function that is guided by the words "true" or "false" since if R is not defined for the selected numbers, then it is false. If these conditions are fulfilled, then R is said to be expressible in **S**.

For example, suppose that you have two natural number n and m and you let R(x,y): x = y. Then for the selections x = 3 and y = 3 you informally know that 3 = 3 via the identity concept. Then this is formally stated in **S** as $\overline{3} = \overline{3}$ and for R(x,y) to be expressible it must be demonstrated that $\vdash_{\mathbf{S}} \overline{3} = \overline{3}$. Then if it is not the informal case that n = m, one needs to show that $\vdash_{\mathbf{S}} \neg(\overline{n} = \overline{m})$. Mendelson (1987), p. 139) shows that such an R is expressible.

For a given number-theoretic relation R, define the "characteristic function" $\mathrm{C_R}$ as follow:

$\mathrm{C_R}(\mathrm{x_1}, \ldots, \mathrm{x_n}) = 0$ if $\mathrm{R}(\mathrm{x_1}, \ldots, \mathrm{x_n})$ is true. $\mathrm{C_R}(\mathrm{x_1}, \ldots, \mathrm{x_n}) = 1$ if $\mathrm{R}(\mathrm{x_1}, \ldots, \mathrm{x_n})$ is false.

As previously stated, the major reason for such expressibility is that, for **S**, a number-theoretic relation is recursive if and only if it is expressible in the language used for **S** (Mendelson, (1987, p. 133)). And since such an expression is in terms of **S**, then, for the universal Turing machine, such a number-theoretic relation R is also expressible in the machine's symbolic language.

Paris and Harrington (1977, p. 1135) have informally proved that the following (*PH*) statement holds for ZFC using the ZFC derivable Infinite Ramsey Theorem. The Finite Ramsey Theorem can be formally established using **PA**. To understand what (*PH*) is stating, requires an excursion into combinatorics.

2. The Paris and Harrington Result.

Recall that the formal axiom systems that yield the same formal theory as Peano Arithmetic are termed, in general, as **PA.** The following illustrates the major combinatorial properties that lead to the Paris-Harrington Theorem, a result not formally provable via Peano Arithmetic **PA**.

1.

Consider the following nonempty set of six symbols that represent natural numbers. $K_6 = \{0, 1, 2, 3, 4, 5\}$.

2.

Let K_n denote a set of natural numbers. Let nonempty $H \subset K_n$ and, in general, $\#H$ means the number of elements in H. Let $\#K_n = n$, where $n \geq 1$. For the first part of this example, let $n = 6$ and K_6 the set in 1. Clearly, $1 \leq \#H \leq 6$. For natural number m, where $1 \leq m \leq \#H$, let the "m-graph," $G(m, H)$, be the set that results from writing down all of the **m-element** sets M, where $\#M = m$, that can be constructed from H.

3.

$G(2, K_6) = \{\{0,1\},\{0,2\},\{0,3\},\{0,4\},\{0,5\},\{1,2\},\{1,3\},\{1,4\},\{1,5\},$ $\{2,3\},\{2,4\},\{2,5\},\{3,4\},\{3,5\},\{4,5\}\}$.

4.

A **P-partition** of $G(m, H)$ is a re-grouping of the members of $G(m, H)$ into subsets containing $P \geq 1$ elements. Let $P = 2$. For $G(2, K_6)$, there are 15 2-element sets that are re-grouped into 2 nonempty subsets, called subdivisions. (In set-theory language, a 2-partition of K_6 is composed of two nonempty subsets A, B of K_6 such that $K_6 = A \cup B$ and $A \cap B = \emptyset$ the set containing no elements.) Below are five examples, where I and II denote subdivisions.

5.

$\mathrm{I} = \big\{\{0,5\},\{0,1\},\{0,3\},\{0,4\},\{2,5\}\big\}$,
$\mathrm{II} = \big\{\{0,4\},\{1,2\},\{1,3\},\{4,5\},\{1,4\},\{2,3\},\{2,4\}\{3,4\},\{1,5\},\{3,5\}\big\}$.

6.

$\mathrm{I} = \big\{\{0,1\},\{0,3\},\{1,3\}\big\}$,
$\mathrm{II} = \big\{$All other 2–element sets.$\big\}$.

7.

$\mathrm{I} = \big\{\{0,1\},\{1,4\},\{1,3\},\{4,5\},\{3,5\}\big\}$,
$\mathrm{II} = \big\{\{2,3\},\{2,4\},\{3,4\},\{0,2\},\{0,3\},\{0,4\},\{0,5\},\{1,2\},\{1,5\},$
$\{2,5\}\big\}$.

8.

$\mathrm{I} = \big\{\{0,1\},\{1,4\},\{1,5\},\{4,5\}\big\}$,
$\mathrm{II} = \big\{$All other 2–element sets$\big\}$.

9.

$\mathrm{I} = \big\{$All other 2–element sets$\big\}$,
$\mathrm{II} = \big\{\{4,5\}\big\}$.

10.

Notice the following facts about these five 2-partitions. For no. 5, let the set $H = \{3, 4, 5\}$. Then $\#H = 3$. Observe that $G(2, H) = \{\{4, 5\}, \{3, 4\}, \{3, 5\}\} \subset$ II.

11.

For no. 6, let $H = \{0, 1, 3\}$. Then $\#H = 3$. Observe that $G(2, H) \subset$ I.

12.

For no. 7, let $H = \{2, 3, 4\}$. Then $\#H = 3$. Observe that $G(2, H) \subset$ II.

13.

For no. 8, let $H = \{1, 4, 5\}$. Then $\#H = 3$. Observe that $G(2, H) \subset$ H.

14.

For no. 9, let (a) $H' = \{0, 1, 2, 3, 4\}$. Then $\#H' = 5$. Let (b) $H'' = \{0, 1, 2, 3, \}$. Then $\#H'' = 4$. Let (c) $H''' = \{0, 1, 2, \}$. Then $\#H''' = 3$. Observe that $G(2, H'),\ G(2, H''),\ G(2, H''')$ are all subsets of partition I. Note that if any $H \subset K_6$ has the property that $\#H > 3$, and $G(2, H)$ is a subset of one of the partitions, then, since 2-element graphs are used, members from set H can be removed, one at a time, to get a set $\underline{H}$ such that $\#\underline{H} = 3$ and the graph $G(2, \underline{H})$ is a subset of the same subdivision.

15.

Now repeat the above process for $K_5 = \{0, 1, 2, 3, 4\}$.

16.

$G(2, K_5) = \{\{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\},$
$\{3, 4\}\}$.

17.

Consider the following 2-partition subdivisions of K_5
$\mathrm{I} = \big\{\{0, 1\}, \{1, 2\}, \{0, 4\}, \{3, 4\}, \{2, 3\}\big\}$,
$\mathrm{II} = \big\{\{0, 2\}, \{1, 3\}, \{1, 4\}, \{0, 3\}, \{2, 4\}\big\}$.

18.

It is not difficult to trace out all of the possibilities like $H = \{0, 1, x\} \subset K_5$ and show that no set $H \subset K_5$ exists such that $\#H = 3$ and such that $G(2, H)$ is a subset of either of these subdivisions. Using the remark in 14, there cannot exist any $H \subset K_5$ such that $3 \leq \#H \leq 5$ and such that $G(2, H)$ is a subset of either of the two subdivisions.

19.

For the, $K_n = \{0, 1, 2, 3, \ldots, n\},\ n = 5$ or $n = 6$, let P denote the number of subdivisions used. What has been demonstrated above?

20.

For $n = 6,\ P = 2,\ m = 2$ and, as illustrated, for each of the five 2-partition subdivisions, there exists $H \subset K_6$ such that $\#H = 3$, and $G(2, H)$ is a subset of one of the subdivisions.

21.

But, for the case of $n = 5$, we, at least, found one of the possible $P = 2$-partitions, where no such $H \subset K_5$ exists such that $\#H = 3$ and $G(2, H)$ is a subset of either of the subdivisions.

22.

All of the results, thus far, depend only upon counting. Consider <u>any</u> distinct six natural numbers $N_6 = \{a_0, a_1, a_2, a_3, a_4, a_5\}$ Then take all the sets previously defined and substitute for 0, a_0, for 1, a_1, for $2, a_2$ etc. All that holds, thus far, for K_6 holds for any N_6. Do the same thing for the five element set K_5. Then all such five-element sets will also satisfy the analysis in 18.

23.

Why are things like this happening? One version of the Finite Ramsey Theorem, as mentioned in Paris and Harrington (1977), can be formally established using **PA**, and shows that the problem is with n. But, what it says is not necessary for what comes next.

There is one additional observation about the partitions 5 - 9. Let h be the smallest number in H. Notice that, in 10 above, the smallest number $h = 3 = \#H$; in 11, $h = 0 < \#H$; in 12, $h = 2 < \#H$; in 13, $h = 1 < \#H$; in 14, for H''', $h = 0 < \#H'''$. But, for any of the sets constructed in 22, if for the smallest number $k \in N_6,\ k > 6$, then, obviously, there is no $H \subset N_6$ that can satisfy such an inequality. So, we have an obvious question. Does the following conjecture hold?

(*PH*) Let $\min(F)$ be the smallest value for the numbers in a nonempty finite set F of natural numbers. For every three positive natural numbers $P,\ m,\ L$, there is a natural number n and a set K_n, where $\#K_n = n$, and, for every

> P-partition no matter how the m-element sets are distributed among the P-partition subdivisions, there always exists a nonempty $H \subset K_n$, where $\#H \geq L$, the $\min(H) \leq \#H$ and $G(m, H)$ is a subset of one of the P subdivisions of $G(m, K_n)$.

The above informal statement (*PH*) can be formalized in **PA.**

Remarkably, Paris and Harrington show that the formalized (*PH*) statement cannot be formally established using PA.

3. Refuting Davies' (DH) Statements.

The types of number-theoretic functions that Turing machines calculate are the "partial recursive functions." A number-theoretic relation R is a "recursive" relation if and only if $\mathrm{C_R}$ is a recursive function.

Recursive functions have special properties. And via $\mathrm{C_R}$ a number-theoretic relation is or is not recursive. The expression "partial function" is used to indicate that its domain need not be the entire class of natural numbers and, here, the term "total" means that its domain is the class of natural numbers. For the function $\mathrm{C_R}$ to be recursive it must be a total function.

For recursive relations, all of the first-order statements using words like "or," "and," "not" etc. can be included. For specific recursive relations, these additional statements produce recursive relations (Mendelson (1987, p. 137)).

Consider Davies' universal Turing machine conjecture as explicitly stated.

> *A universal Turing machine has as its axioms all of the physical laws. The machine produces or, at the least, predicts each physical event since each is $\mathcal{U}$-computable. Hence, the universe lends itself to simulation via such a machine. (It is a philosophic stance whether a physical event is the result of the physical laws).*

This hypothesis has no meaning unless additional somewhat doubtful requirements are added. Turing machines operations are completely related to a language. One needs, at the least, to assume that (1) each physical property that characterizes entities and events within our universe can be expressed using a symbolic language $\mathcal{L}$ that is based upon a finite alphabet. (2) The relations that exist between the properties that govern our universe are expressible in $\mathcal{L}$. (3) Any physical event produced by application of the physical laws is completely describable via expressions in language $\mathcal{L}$. (4) There are no describable events that are not produced by physical laws as they are so expressed. The describable physical laws themselves are not classified as describable physical events. These restrictions are rather obvious. The term $\mathcal{U}$-computable means universal Turning machine computable. Thus, this article is restricted to the following meaningful statements.

> (DH) *Under assumptions 1 - 4, a universal Turing machine has as its axioms all of the physical laws. The machine produces or, at the least, predicts each physical event since each is $\mathcal{U}$-computable. Hence, the universe lends itself to simulation via such a machine.*

If any of these four assumptions is not accepted, then the original Davies statement needs to be restated. In this case, it is:

> (DH) *A universal Turing machine has as its axioms all* <u>*describable*</u> *physical laws. The machine produces or, at the least, predicts each* <u>*describable*</u> *physical event since each is $\mathcal{U}$-computable. Hence, the* <u>*describable*</u> *universe lends itself to simulation via such a machine.*

However, (DH) is based upon unverifiable assumptions. One is that all of the correct physical laws can be stated via a specific comprehensible language $\mathcal{L}$. That there are no physical laws that we cannot comprehend via our senses. A remarkable philosophic stance to take. Of course, I am not sure how we would know that we have actually formed a collection of all such descriptions.

What is an appropriate algorithm? "An algorithm is an explicit set (collection) of instructions for a computing procedure (not necessary numerical) which may be used to answer any of a given class of questions" (Hamilton, (1978, p. 152)). For a Turing machine, it has been shown exactly what algorithms it can follow. They are of one and only one type of expression formed using **S** arithmetic. But, the results in this article are independent from this general concept of what constitutes an algorithm

In the following portion of this article, unless otherwise stated, when the phrase "physical law(s)" is used it means describable in a Turing machine language. When the phrase "physical event(s)" is used it means a describable physical event in terms of the language used to describe the physical laws, with possible additional new terms for physical entities or processes characterized by parameters consistent with the physical law parameters.

> Let S be "a description for physical event Q and the event Q occurs," R be "the universal Turing machine computed result is Q," and P = "the physical laws." Since Turing computation uses physical laws, then the basic Davies statement is $((\mathrm{P} \to \mathrm{S}) \to ((\mathrm{P} \to \mathrm{R}) \to \mathrm{S})))$. It is assumed, as fact, that finite collections of the collection of all physical laws are appropriately expressed in the language of specific Turing machines and such machines compute (i.e. predict) physical events (i.e $\mathrm{P} \to \mathrm{R}$). Thus under this fact for $((\mathrm{P} \to \mathrm{R})) \to \mathrm{S})$

to be true, S is true. To verify the "truth" of an implication A $\rightarrow$ B, one assumes that A is true and establishes that B is true. Hence, ((P $\rightarrow$ R)) $\rightarrow$ S) $\rightarrow$ (P $\rightarrow$ S) is a true implication, although, for this implication, the "truth" still follows even if you substitute any proposition for P in (P $\rightarrow$ S). Thus, under the requirement that collections of physical laws act as "axioms" for the universal Turing machine, the following also holds for the (DH). If the collection of all physical laws, predicts a physical event Q and Q occurs, then this event is U-computable. If an event Q is $\mathcal{U}$-computed via a Turing machine, and it occurs, then it is produced by describable physical laws.

For the class $\mathcal{Q}'$, finite applications of the prime operation and the defined operations $\oplus$, $\otimes$ satisfy the axioms **S**. Such a procedure using class terminology is considered as valid (Jech, (1971)). Using the relation $\mathrm{R}(\mathrm{x},\mathrm{y}) : \mathrm{x} < \mathrm{y}$, there is a basic characterizing property for $\mathcal{Q}'$ that corresponds to ZFC object ω. It states that there is some y such that for each x that corresponds to a member of $\mathcal{Q}'$, $\mathrm{x} \in \mathrm{y}$ (i.e. $x < y$). This relation satisfies the requirements for $\mathcal{U}$-computation. The corresponding restricted formal statement is $R\#(x,y): \ (x \in \omega) \wedge (y \in \omega)) \rightarrow x < y$. The number-theoretic relation Pf(p,q), where q is a Gödel number for a wf, determines whether there is a proof using **S**# that yields as the last step a wf. The relation satisfies the requirements for $\mathcal{U}$-computability Let q be the Gödel number for $R\#(x,y)$. However, for every natural number Gödel or not $\mathrm{C}_{\mathrm{Pf}}(\mathrm{p},\mathrm{q}) = 1$ since in the formal expression $y \in \omega$ is also one of the x, and yields the statement $y < y$. This contradicts the assumed consistency of **S**#. (See "Proof" after Appendix.) In order to maintain consistency, a universal Turing machine cannot predict via numerical computation the existence of the mental image $\mathcal{Q}'$.

If the specific mental intention that yields $\mathcal{Q}'$ is produced by a collection of physical laws, then this mental physical event is not $\mathcal{U}$-computable. If this mental intention is not produced by physical laws, then it is still not $\mathcal{U}$-computable.

Roger Penrose and others have mounted arguments against the Davies' (DH) based upon the human brain and quantum theory. Others have countered with other theoretical possibilities. Penrose's original conclusion is related directly to the Halting problem for Turing machines "The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics," (1989)). "Roger Penrose argues that known laws of physics are inadequate to explain the phenomenon of consciousness. He uses a variation of the Halting theorem to demonstrate that a system can be deterministic without an algorithm." [WS-3]

There are physical arguments for and against this Penrose stance based upon theoretical statements from quantum physics. The most recent ones are pure speculation that have, at present, not been physically verified. They do generate published articles, however.

> What are mental intentions for the construction of the class $\mathcal{Q}$? It appears that it is an "idea" that I am following a rule for construction with a final intent in "mind." I do not seem to remind myself of this intent. In the illustrations in the Appendix, I mentally construct and, due to the unbounded view, this procedure will suddenly be mentally perceived as the completed infinite $\mathcal{Q}$ view. **That is, a procedure that so readily implies that the $\mathcal{Q}$ view is that of a completed infinity.** The white road and panes simply appear within the $\mathcal{Q}$ view, and the view mentally displays an object that has the properties of a complete infinity. I base this mental appearance only upon a form of "observation." The experimental aspects are the same as interpretations given by a human being from personal physical observations or observations of a machine's output. They are exactly how physical scientists determine the outcome of an experiment. Thus, to establish that there is a mental process going on that is not Turning computable requires a preponderance of evidence that others can more or less automatically obtain the $\mathcal{Q}$ view of the white road with the panes via such a mental intention. This appears to be the case.

Turing machines are idealized computers. A Turing machine is defined as a finite set of tape descriptions or action tables (Mendelson, (1987, p. 232). These determine what a Turing machine can "type out." Peano Arithmetic and the first-order predicate calculus determine the computations that any Turning machine performs. Also they use a potentially infinite tape (Kleene,(1967), Mendelson, (1987)). One needs to use certain intuitive concepts such as what the term "finite" signifies. A Turing machine presented statement is a finite string of alphabet symbols constructed form a finite alphabet. To show that Turing computing and **PA** have the same properties the alphabet symbols are replaced by a finite set of natural numbers and these can be further replaced by special tape symbols. For Turing machines, a Gödel type coding (Mendelson, (1987, p. 246)) is used for the Turing machine's alphabet symbols, as representations for its processes, as representations for natural numbers and, even, Gödel numbers for the name of a machine and its computations (Mendelson, (1987, p. 232)). Such a coding is also used for a formal **PA** language and the first-order predicate calculus for its "computing" processes.

What is a universal Turing machine? It is a machine based upon the an algorithm that "enumerates" the Turing machines. That is, in a potentially infinite way there is an algorithm that names the individual Turing machines. "However, we can encode the action table of any Turing machine, in a string. Thus we can construct a Turing machine that 'excepts' on its tape a string describing the input tape, and computes the tape that the encoded Turing machine would have computed." " [W]ith this encoding of actual tables as strings it becomes possible in principle for Turning machines to answer questions about the behavior of other Turing machines. Most of these questions, however, are undecidable, meaning the function in question cannot be calculated." [WS-2].

The informal (*PH*) has a formal $\mathcal{T}(\mathbf{S})$ expression $\mathcal{PH}$. It has a Gödel number q, and, hence, the Gödel (non-zero) number h(q) for $\mathcal{PH}\#$. The recursive number-theoretic relation $\mathrm{Pf(y,x)}$ relates the Gödel number x of a wf with Gödel number y of its **S** proof if any. Substituting informal natural numbers into this number-theoretic relation, if the relation is satisfied, then it is informally true. Otherwise, it is false. The relation Pf is a recursive relation. To state that (*PH*) is not provable using **S** is equivalent to stating that $\mathcal{PH}\#$ is not provable using $\mathbf{S}\#$. Thus under the recursive correspondence, let q be the Gödel number for $\mathcal{PH}\#$. Although $\mathcal{PH}$ is provable in ZFC, it is not a theorem in $\mathcal{T}(\mathbf{S}\#)$. Thus, for each informal natural number y, $\mathrm{C_{Pf(y,q)}} = 1$. Thus the fact that $\mathcal{PH}$ is provable is not $\mathcal{U}$-computable.

> The statement (*PH*) is formalizable in $\mathcal{T}(\mathbf{S}\#)$. But, the Paris-Harrington proof that (PH) holds uses informal set-theory and is a last step in a ZFC proof. Such a proof is formalizable using ZFC and is physically expressed on published pieces of paper (Paris-Harrington (1977, p. 1135)). According to (DH), there are physical laws that produce this physically presented object. But, the fact that a physical event, the proof, exists is not $\mathcal{U}$-computable. This falsifies (DH). If the proof of (PH) is not the product of physical laws, then this would also falsify (DH).

For any fixed m, the informal statement $\mathrm{PH_m(P,L)}$: "for each two positive natural numbers P, L, there exists an n such that . . . ." has a formal proof using $\mathbf{S}\#$. Hence, for the corresponding PH_m, $\vdash_{\mathbf{S}} PH_{\overline{m}}$. The function $\mathrm{C_{PH_m}} = 0$ for each L, P. The function $\mathrm{C_{PH_m}} = 1$, when either L or P is zero. If $\mathrm{PH_m(P,L)}$ is recursive, then these values for $\mathrm{C_{PH_m}}$ are $\mathcal{U}$-computable. Since in all cases only finite sets of numbers are being considered, an actual trial and error process, as slightly indicated in the (*PH*) illustrations, has been applied to discover the H for specific cases. The n is often large. Of course, physical laws allow for this calculation. For any fixed selection m and any P and any L, this actual computation is a type of repeated experiment. Under the recursive requirement and (DH) and for each of the selected P and L, the physical laws have not changed, and the $\mathcal{U}$-computed result is obtained that there is a proof for each m-specific case.

It seems appropriate that at this point that the human computer apply inductive generalization to the finite collection of written proofs as based upon another concept "the uniformity of nature." Then a paper is published asserting that **based upon known physical laws for each P, m, L a proof exists**. But, these two employed concepts are not actually physical laws. They are statements about the physical laws. So, although after a trillion of such experiments a universal machine might be instructed to stop and then express the inductive generalization, the statement itself has not actually been obtained via a recursive number-theoretic relation. **If these forms of falsification are not accepted, then the $\mathcal{Q}$ or $\mathcal{Q}'$ and (PH) do lead to cosmological falsifications.**

As discussed next, noticing examples such as K_5 and $K_{,}6$, and although much more difficult to imagine, such collections of collections of space configurations exist. And then for an infinite volume space, the collection of all such configurations should exist. Hence, it is likely that (*PH*) inspired physical configurations falsify (DH).

Although there are expressions that are not $\mathcal{U}$-computable, this says nothing about the number-theoretic functions used throughout physical science.

4. Cosmological Falsifications of (DH).

In what follows, analogue models are presented. These are models for behavior. Once the behavior is rationally justified, then other objects can be substituted for a model's constituents.

Are there physical laws that yield physical events that are not $\mathcal{U}$-computable and they do not actually correspond to events that are but the products of mental activity? Yes, if one accepts that physical laws include parameters that upon substitution into the law describe different types of universes. The law is The Law of Physical Metrics, the Law of Space-Time. This states that the general gravitational behavior of our universe is controlled by space-time metrics that satisfy the *Cosmological Principle* (Lawden (1982, p. 171)). Entities are introduced, which include observers, and, from the Cosmological Principle, cosmical (cosmic) time is defined. Relative to cosmic time and the adjoining of additional axioms that define metrics, the simplified geometry applied describes "space" in terms of positive, zero, or negative curvature. The three metrics obtained satisfy the Cosmological Principle (Lawden (1982, p. 177)).

The Hilbert-Einstein gravitational field equations as a physical law are adjoined. The overall generalized and approximating view for entities in the universe is that they form a perfect fluid that satisfies a general four-term metric statement that contains, for various coordinate (measuring) systems, one variable that represents cosmic times. From the coefficients of the Robertson-Walker metric, expressions for the energy-momentum tensor are obtained and expressions for cosmic dynamics derived. For the appropriate axioms, there are computer programs that can produce the derivations and, hence, they are all $\mathcal{U}$-computable.

All aspects of Turing computing are related to coded numerical expressions. Relative to this, the only "numbers" it can predict, via repeated symbolic representation (Mendelson (1987, pp. 246)), correspond to a fixed finite collection of Gödel coded symbols. A physical interpretation for the result that a number is computable or that a number cannot be computed is external to the actual numbers that are or are not $\mathcal{U}$-computable. The most basic idea is that a physical event and behavior correspond to symbolic expressions. Thus what such a symbolic expression means physically depends upon an interpretation. There are three general types of universes that satisfy the Robertson-Walker metric. For two of the three universes and from the Expansion of Space and assuming there is a moment when expansion began, Felder gives an interpretation of the predicted results.

> "The total size of the universe has not changed - it's still infinite - but the volume of the space containing any particular group of galaxies has grown because the separation between galaxies is now larger." Then Felder states that at the approximate moment when expansion began, "If the universe is infinite, then it was also infinite at that early time. The density was enormous and the distances between particles vanishingly small, but that dense mass of particles went on forever." Felder also states that is appears more likely than not that our universe is infinite. [WS-1]

I suppose "at that early time" means as measured from now to a moment when expansion began. The "particles went on forever" refers to the requirement that there be "infinitely" many such objects. Thus, quantum fields "go on forever" and, unless, produced by something else that has existed "forever" they have existed forever. I suppose this is in cosmic time. If one continues such reductionist thoughts, a logical regress may develop; not a useful result.

There is a controversy as to just what one means by a single infinite universe or an infinite collection of universes. Do we mean a completed infinity or the potentially infinite? It does not seem very productive and leads to confusing mental images to consider the term "curvature" as a true geometric term. The Riemannian geometry used is a generalization of three dimensional geometry. But, as pointed out by Wheeler and others (Patton and Wheeler (1977)), this is a faulty comparison. Geometric language is used but it needs to be translated into actual physical behavior relative to both three dimensional space and cosmic time. The translation is usually done by discussing the behavior of gravitating objects and the paths they follow if allowed to freely move through a gravitational field. This movement is compared to that of an almost constant gravitation field in a local fixed laboratory. One does not need to consider any of the confusing geometric descriptions relative to "geometry." Predicted physical behavior is sufficient.

In what follows, the term "object" or "entity" is used generically for the various notions that constitute its use, such as "excited states of quantum fields." Further, at a moment in cosmic time and depending upon whether a universe is a "finite" or "potentially infinite" object universe, the concept of a finite "number" of such objects within a universe at any moment in cosmic time is also used generically as a quantity equal to or less than a maximin or, for the potentially infinite case, a number increasing as cosmic time progresses, respectively. Within any actual physical universe, an algorithm is used to obtain space configurations. Relative to expansion there is the idea of the "open" or "closed" universe.

In order to model the basic types of expanding universes, it is assumed that expansion is being observed during a period of cosmic time. In the most basic equations, there is a critical density of objects that determines whether the universe is open or closed. If one includes the notions of "Dark Energy" and "Dark Matter," then the present estimate of the critical density is that it is equivalent to 5 atoms per cubic meter. The actual observed density is about 2 atoms per cubic meter. This implies that our universe is open. This means, however it is produced, that the "expansion" will not cease in cosmic time. That is, from the moment that it was declared that our universe is open, observations over increasing intervals of cosmic time will continue to display this behavior. But, does this mean that our universe is composed of infinitely many objects (at any moment in cosmic time there is a collection of distinct objects each of which corresponds to a member of $\mathcal{Q}'$ and conversely) or that it is a potentially infinite object universe or a maximum finite object universe?

How can we have a slightly more accurate view of the notion of a universe where numerical expressions represent the names for entities that comprise a galaxy? At the end of the material in the Appendix a solution to this "mental" problem is given. Simply remove the white road relative to these views.

The Cosmological Principle allows a general pattern for universe behavior to be exhibited by considering its appearance as viewed in one direction. All that is needed in the following behavioral illustrations is that "snapshots" be given for a few moments in increasing cosmic time.

Consider objects in $\mathcal{Q}'$ abbreviated by their numerical names. That is, $\emptyset = 0,\ \emptyset' = 1,\ \emptyset'' = 2,\ \emptyset''' = 3$, etc. Let the ZF set $\mathcal{F}(\mathcal{Q}')$ denote the collection of all finite subsets of $\mathcal{Q}'$. Then consider the ZF set of $\mathcal{T}(\mathbf{S}\#)$ class $\mathcal{F}(\mathcal{Q}') \cup \mathcal{Q}' = E$. Below are sequences, partial or otherwise, defined on intervals or on $\mathbb{N}$ that relate members of $\mathcal{Q}'$ to members of E, respectively. These are considered as prototypes for the behavior of a universe relative to various cosmological laws that include expansion. Sequence values taken from $\mathcal{F}(\mathcal{Q}')$ indicate gravitationally bound physical entities. The distinct values taken from $\mathcal{Q}'$ represent volumes of space. The values themselves are not to be considered as a numerical measure for the volume. The number of such values and the order in which they appear illustrate the expansion of space. In each displayed case, the indicated rule is to be repeated at all moments of cosmic time after the ones displayed. The rule can easily be expressed via ZFC set-theoretic notation and the results form a set or class. The observer is at 0.

An Expanding Universe.

(I) A maximum number of physical objects and space creation.

$$f_1[[0,7]] = \Big\{0, 1, \{1,2\}_2, 3, 4, \{3,4\}_5, 6, 7\Big\}.$$

$$f_2[[0,9]] = \Big\{0, 1, \{1,2\}_2, 3, 4, 5, 6, \{3,4\}_7, 8, 9\Big\}.$$

(II) A potentially infinite number of physical objects and space creation.

$$f_1[[0,7]] = \Big\{0, 1, \{1,2\}_2, 3, 4, \{3,4\}_5, 6, 7\Big\}.$$

$$f_2[[0,9]] = \Big\{0, 1, \{1,2\}_2, 3, 4, 5, 6, \{3,4,5,6\}_7, 8, 9\Big\}.$$

(III) A maximum number of physical objects within an infinite (space) volume.

$$f_1[\mathbb{N}] = \Big\{0, 1, \{1,2\}_2, 3, 4, \{3,4\}_5, 6, 7, \ldots\Big\}.$$

$$f_2[\mathbb{N}] = \Big\{0, 1, \{1,2\}_2, 3, 4, 5, 6, \{3,4\}_7, 8, 9, \ldots\Big\}.$$

(IV) A potentially infinite number of physical objects within an infinite volume.

$$f_1[\mathbb{N}] = \Big\{0, 1, \{1,2\}_2, 3, 4, \{3,4\}_5, 6, 7, \ldots\Big\}.$$

$$f_2[\mathbb{N}] = \Big\{0, 1, \{1,2\}_2, 3, 4, 5, 6, \{3,4,5,6\}_7, 8, 9, \ldots\Big\}.$$

(V) Infinitely many physical objects within an infinite volume.

$$f_1[\mathbb{N}] = \Big\{0, 1, \{1,2\}_2, 3, 4, \{3,4\}_5, 6, 7, \ldots, \{i,j\}_n, \ldots\Big\}.$$

$$f_1[\mathbb{N}] = \left\{0, 1, \{1,2\}_2, 3, 4, 5, 6, \{3,4\}_7, 8, 9, \dots, \{i,j\}_n, \dots\right\}.$$

For the next results, an infinite volume universe is required and infinite volumes exist within such a universe. It is not necessary to consider what comprises an infinite space such as quantum fields or other proposed entities relative to physically interacting objects. Whether one accepts the notion of the creation of space or not, an infinite volume is a viable physical law and it rationally predicts configurations that falsify (DH). A law of gravity, Hilbert-Einstein's or Newton's, predicts the accepted physical law - Kepler's Second Law of planetary motion. Recall that this law states that a space-configuration, not marked out by physical matter, has the same area over a fixed time period as a planet ideally moving along its path of motion around a gravitating object, like the Sun. As with this law, one can consider space configurations.

> The human brain has developed through application of physical laws. Or, at last, its composition is consistent with such laws. Nature behaves in certain ways. We use models or mental constructs that replicate this behavior. Do the line segments physically exist? Observationally, the answer is no. It is assumed that there is "something" in Nature that corresponds to the behavior that we represent this way. For me, the "something" need not be physically defined. This "something" is present before Kepler discovered the relationship. All space configurations that relate to physical behavior <u>represent</u> "something" in Nature that corresponds to the configurations.

Using the Cosmological Principle, the general behavior of our universe can be determined by viewing in one direction. So, consider such an observer. The space configurations are the rectangular regions outlined by the twelve edges of each pane for the $\mathcal{Q}$ view. Now view the panes as if there is no apparent distance between them. Consider doing the same thing but viewing $\mathcal{Q}$ as contained in a night sky at an isolated region so that there appears to be no boundary preventing the panes from forming a completed infinity. This view yields a completed infinity of such configurations embedded into an infinite volume universe. This is view $\mathcal{Q}_0$ and it corresponds to $\mathcal{Q}'_0$. **The physical space they represent physically exists in an infinite volume universe.**

In order to characterize this space configuration, it is in one-to-one correspondence with the formal ω of ZFC. To establish a universal Turing machine cannot numerical determine the existence of the complete infinite physical object represented by $\mathcal{Q}'_0$ exists in the infinite volume universe consider the method previously used for the case of $\mathcal{Q}'$. This method yields that this physical event is not $\mathcal{U}$-computable relative to a numerical representation for its existence.

There is a physical law that states that our universe is of infinite

> volume. The physical existence of a completed infinite collection of such configurations within such a universe is a direct result of this physical law. However, the fact that this physical law yields this configuration, a physical event, is not $\mathcal{U}$-computable.

The last conclusion is strengthen if the space configurations are related to other physical objects and the behavior of these objects can be described in terms of these configurations. An infinite volume universe necessarily contains the volume physically occupied by the objects and collections of the physical entities that separate one from another. This holds even if there are but finitely many such physical objects. For such physical collections, the completed infinity of such configurations can be placed in such a manner that the expansion produces physical entities that move or are altered in number relative to these actual space configurations. For the panes view, the "particles" simply move or are altered within the panes via "snapshot" views.

> There is a completed infinity of configurations that satisfy the described physical behavior that results from the motion associated with the separation occuring between collections of physical objects due to the physical law of expansion. Thus, although an accepted physical fact, a universal Turing machine can not predict, for such physical objects, this physical object behavior in terms that are relative to the entire collection of objects as they are embedded into an infinite volume universe. That is, such behavior is not directly $\mathcal{U}$-computable.

Thus, if an infinite volume universe is not rejected as an impossibility, then there are, at the least, two physical events that are not $\mathcal{U}$-computable and this yields physical falsifications for the Davies' (DH) hypotheses. As mentioned, there is a third falsification, the (PH) inspired physical configurations that also falsify (DH).

Appendix

Examples of How to Imagine the Infinite.

Some people who want to pollute our world with some rejected late nineteenth century concepts claim that what I present here does not exist. They insist that human beings are like them. They insist that human beings are incapable of imagining the "infinite." This claim is utterly false.

What does the term "completed infinite" mean? As you'll see shortly, the notion of the "potential infinite" is rather easy to imagine. A completed infinite would be "something" that "contains" a potentially infinite "something." So, how do I mentally image a completed infinite? As you will notice from the following two descriptions, I cannot successfully draw this mental image on a piece of paper. But, it is shown that properly described mental images will coincide with the concept of the (completed) infinite.

An Infinite White Road; the Beginning of Various Completed Infinities.

Close your eyes or go into a “completely” dark room. That is, in some way, remove the visible light from your view. Now imagine (mentally image) a white road **with an assumed fixed width** beginning at a position within this black background. There is a straight line segment where the road “starts” in the black background. The entire road is surrounded by the totally black background. The edges of the road are straight lines and, from your mental view, the road appears to extend towards the “upper right.” The road appears to be slowly dwindling in width until it appears to be just one single dot. This is a “vanishing point.” However, you still known that the road has a fixed width. From your experience with actual roads, you conclude that the road extends further and further from its start position. This background has no “visible” boundary. (This fact is what yields a meaningful vanishing point.) But, notice that even for the unbounded black background, the road still extends in a direction relative to your mode of viewing.

What actual procedure has been done to invoke these and the other images to be described later?

> I have presented, in word form, a description that, via the concept of ”mental intentions,” should be perceivable. I slowly read the words and then apply the process. They have meaning based upon my previous experiences. I am mentally aware of my mental voice, the ”speaking to myself voice.” The images are recalled. They are not of the same character as they would be if I were viewing with my vision. Indeed, for me, they are rather not related to my vision since I can superimposed various ordinary images over my visual images without any interference. But, now I command them to appear in a place for which I have no experience, within a completely black region. It is a situation that does not correspond to any physical laws with which I have had experiences or know about. The images appear in the form I demand. I have not given any further description as to how my mention intention is to produce the desired result. The requirement is that what I read or ”mentally speak” is to be carried out and then, suddenly, I mentally perceive a completed road, a completed infinity the corresponds to my mental intention. The images mentally appear as I have requested. This ”request” is a deterministic notion, but it determines or produces the complete outcome without an algorithm.¡p¿

This form of imaging is not related to an external brain probe that can produce mental images. This is an internal "command" or "an internally stated intention" the produces the images and the final result. According to Eccles and Robinson (1984) and others, if I intended to perform certain actual repeated physical motions, then there is no physical connection found between certain mental intentions and the actions performed. But, in this case, the actions are all related to internal mental actions that yield internal mental results. It is of little significance if there is speculation that a "such and such" describe physical process "could" produce, from a type of mental command, such mental actions - the forming of mental images. Only predictive and observable experimental results based upon stated physical laws can overcome statements like "there is no physical connection" or "that no actual physical processes have been shown to exist that predict these mental outcomes." Indirect verification for assumed entities and their behavior only implies a possibility and does not indicate fact.

The potential infinite is a concept relative to the step-by-step notion, step-by-step actions, instructions that are carried out in a step-by step manner. It is a myopic restricted local view. As to operations with numbers, it states that one can apply operations only in a step-by-step manner and only applies the "intuitive" idea of the finite. That is, it is the simple counting concept. It corresponds to a specific axiom system that is defined only in terms of operations performed on finite collections of objects. You never look at the objects as a total collection. You start at the beginning of the road. You are on it and walking. The potentially infinite is the idea that you can take one step after another. But you are always looking at the road just a step ahead. You never cease this view. All you know is that you can, with this restricted view, continue to take that step. The distance you have covered is finite and you can add to this distance. Although, you continue to walk, you give no thought as to ever being able to stop.

Returning to the original image, unfortunately, there is one aspect I cannot draw. In my mental image, there is no edge to this black background. There is such an edge controlled by an individual's field of vision when physically "seeing" any collection of objects such as an actual road moving off to the upper right in ones visual-field. Further, any finite drawing must have an edge. So, what part of this description is the completed infinite?

It is your view that there is an unbounded black "something" that contains the entire road. It is your unbounded view that is a mental model for the complete road. You are viewing the completed road. One might claim that there is "nothing" in the black background. But there is. Your viewing stance is in it. It's from this background that you are viewing the entire road. From this view, the road is conceptually complete. In such cases, you can't rationally view yourself as part of the image. For if you could, then this leads to a logical regress and your brain would close down.

[The logical regress occurs this way. If I can image myself viewing this scene, then I can imagine myself imagining myself viewing this scene. Then I can mentally image myself imaging myself imaging myself viewing this scene. Etc.]

Imagining more Complex Completed Infinities.

Close your eyes or go into a "completely" dark room. That is, in some way, remove the visible light from your view. Imagine a glass or other type of square pane or plate that is placed perpendicular to the road and has the same width as the road. The panes are vertical and placed one behind the other but with a fixed distance between them. This is done so that, from your view of the panes, you can "see" a slim rectangular region across the top of each pane. You only see the first pane completely. For the other panes, you see the top and a piece of the right-hand side. The panes do not change in width but retain the same width as that of the road. The space between them appears to be slowly dwindling as the panes appear to get less wide. From your experience in viewing such events, this is what you expect from a view of this configuration if it takes place from various positions **within** the view itself. Thus, the panes "appear" to get smaller and smaller in size until they appear to be just one single dot. From your mental view, they appear in a row that extends towards the "upper right." They extend towards a "vanishing point."

The fact that the background has no boundary and the black surrounds this row of glass panes is what, as described below, yields a meaningful vanishing point. (You might have to color the panes to imagine this successfully.) Although this black background has no "visible" boundary, you still have a direction relative to your mode of viewing.

As you start from the closest pane and move mentally to the smaller and smaller panes, you mentally notice that there is another pane "after" the one you stopped at. For this case, this is the notion of the potential infinite.

As before, there is one aspect of the image I cannot draw. This is the actual aspect that, for me, yields a model for a completed infinite collection of panes. In my mental image, there is no edge to this black background. Nothing is there that seems to stop the panes. There is such an edge controlled by an individual's field of vision when physically "seeing" any collection of objects moving off in the same visual direction. Further, any finite drawing must have an edge. So, what part of this description is "the pane" that includes all the other panes?

It is your view and the entire conceptual mental image you have of the panes. You are viewing it from where? It's within this "view" that each of the panes exist. It's within this "view" that these images occur. All panes mentally exist within this view and this "complete" view can be defined as "something" that certainly contains each pane. So, what do you have? If this "view" did not mentally exist, then what are you conceptually imagining? Using this obtained complete mental view, one has a model for the completed infinite. Thus the black background can be thought of as the "something" through which you are viewing the entire road and the panes.

Except for the first pane, as you view a particlar pane, then there are those that "came" before the one you are viewing. A pane and the finite collection of the panes or objects that came before is also a way to view the potential infinite.

Here are objects that yield a third and forth infinity that is controlled by the original one. Consider finite set of rules that you can follow that produce an object. You follow the same rules each time an object is produced and attach the object to a pane. Of course, the objects attached to the panes could be identical. But also, each object can be identical but with distinct "numerical" name. The "front" of a pane is defined as the surface you can mentally perceive from the start line if finitely many of the "previous" planes are removed. (Notice how the items are given in terms of finite stuff that exists or finite collections that have already been produced.) You then either use what you have constructed thus far to construct a new pane object or apply a fixed rule relative to or not relative to the other panes to construct an object on a pane. So, construct the objects one-at-a-time and glue the first object to the "front" of the first surface. Then as the objects are completed, glue them to each successive pane. In this imagined scenario only sequential behavior is being considered. There are no "time" considerations. Each and every pane has a glued object. ¡p¿

> **Notice that I have not given an algorithm, a finite description, for any processes that yield the "view" from the written description. I base this result only upon a form of "observation."** The experimental aspects are the same as the interpretations given to physical observations by a human being from personal observations or observations of a machines output. Thus, to establish that there is a mental process going on that is not Turning computable, requires a preponderance of evidence that others can also imagine this same view. As mentioned, it is of little significance if there is vast speculation that "such and such" described physical processes "could" produce such actions or such images. Only predictive experimentation based upon stated physical laws can overcome the above statement that there is "no physical connection." Indirect verification for assumed entities and their behavior only implies a possibility and does not indicate fact.

It seems that there is, at least for me, a mental model for the class of glued objects; another completed infinity. Is there a fourth completed infinity that can be viewed relative to the objects and panes? Just write on the road immediately in front of the pane (between the panes after the first one) the number for the constructed object. Since we have the view that gives us the natural numbers, then this is possible. Indeed, finite collections of symbols written left-to-right with repetition allowed correspond to unique natural numbers. Thus you can write any distinct combinations in place of the numbers. So, at one time we have a view of four completed infinities; the road, the panes, the glued objects, and finite and distinct sets of symbols.

There is, however, one more highly significant view. Once the panes and any objects that may be attached "appear" in your view, then the white road can be removed. The view of the of panes and any objects on them continues to be a view of a completed infinity.

Proof

The C-Rule (Mendelson, (1987, p. 64-66))is applied in the following. The reasons for each step can be found in the section on Formal Number Theory in Mendelson, (1987, pp. 116-129)). The formal contradiction, step 9, contradicts the consistency of PA. Hence, in any model for PA, $y < y$ is false.

$$(1)\ y < y.\ (2)\ \exists w((w \neq 0) \land (y + w = y)).\ (3)\ (b \neq 0) \land (y + b = y).\ (4)\ y + b = y.$$

$$(5)\ b \neq 0.\ (6)\ y + 0 = y.\ (7)\ y + b = y + 0.\ (8)\ b = 0.\ (9)\ (b \neq 0) \land (b = 0).$$

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