

$E = mc^2$ is Not Einstein's Discovery

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1. Introduction

It appears that some scientists have not received the proper credit for significant discoveries for which they have priority. However, without specific and irrefutable information, it is not possible to give convincing reasons why these individuals are denied recognition and why others are given credit for their scientific discoveries. In 1996, I was asked whether Einstein or Hilbert originally formulated certain aspects of General Relativity. (Hilbert presented the gravitational equation(s) prior to Einstein.) The questioner said that he knew very little about Einstein's achievements except for such things as " $E = mc^2$." I answered his question relative to the Hilbert verses Einstein controversy but neglected to discuss the more easily explained $E = mc^2$. What follows shows exactly who developed the **idea** that "radiation" can be characterized as having an apparent mass and that it is not Einstein in his 1905 paper. Except for the last remarks on Olinto De Pretto, this article is concerned mostly with "radiation" and its relation to $E = mc^2$.

2. H. Poincaré

In a 1920 (16) and repeated in a 1933 publication (17), W. Pauli (1945 Nobel - Physics) wrote relative to an example he gave that depends upon the notion of the "momentum of radiation" that "This example is especially of interest, because Einstein with its help derived for the first time [(1905) paper referenced] the principle of the inertia of energy as a universal law." As is shown below, this statement relative to Einstein's 1905 paper is false.

For this physical scenario, the actual meaning of the $E = mc^2$ is that it represents the apparent mass gained or lost under absorption or emission of radiant energy of value E . Jules Henri Poincaré (1854-1912), in 1900 (20), considered an expression for what he calls the "momentum of radiation" M_R . It is $M_R = S/c^2$, where S represents the flux of radiation and c is the usual velocity of light. He gives an example, using the conservation of momentum in a recoil process. He uses m for the mass of the recoiled object and v its velocity. He then uses $S = Ec$, where E is the energy of radiation. He lets $E = 3 \times 10^{13}$

* Apparently, this revision was not used to replace the previous one on the date indicated. This error has been corrected.

ergs, $m = 10^3$ grams, and c be its usual value and calculates that the recoil velocity must be $v = 1$. This Poincaré calculation is expressible in the form $mv = (E/c^2)c$. From the viewpoint of unit analysis, E/c^2 takes on the role of a “mass” number associated with radiation.

A way to arrive at this relation, by means available at that time, is application of the action-reaction principle and the basic Maxwell formula $F = (1/c)dE/dt$ for the force exerted on an absorbing body caused by receiving electromagnetic radiant energy at the rate of dE/dt . Relative to the object reacting to this energy, one also has that the force is the rate of change of the momentum added to the object. By the conservation of momentum, this is equal in magnitude, if not yet having significant physical meaning, to the momentum associated with the radiation. Thus $dM_R/dt = (1/c)dE/dt \Rightarrow M_R = (1/c)E + C$, and, under the obvious initial condition in the problem, $C = 0$. Hence, $M_R = S/c^2$, where $S = Ec$.

Essentially embedded in this Poincaré expression is a fundamental radiation result. Since $M_R = (E/c^2)c = E/c$, where E/c^2 is a “mass” term, it might immediately be claimed that the radiation has an “apparent mass” m_r . Under this claim, using $M_R = m_r c$, then $m_r c^2 = E$.

Poincaré specifically recognizes this association, but associates it with a possible different effect rather than with an actual “mass” increase or decrease for the object in question. His view is that for “Electromagnetic energy . . . behaving like a fluid endowed with inertia, one must conclude that if any apparatus whatever, after having produced electromagnetic energy, sends it by radiation in a certain direction, the apparatus must recoil like a cannon which has launched a projectile” (20). This is not exactly correct since it’s based upon a 1900 point of view. This Poincaré statement needs a slight modification since any expression that equates M_R to anything must depend upon the application of the radiation. The derivation given above is relative to the force on an “absorbing body.”

If certain accurate experiments conducted from 1887-1902 on the photoelectric effect were known earlier, then for such scenarios Poincaré may have noticed that recoil is not the effect and maybe he might have speculated relative to kinetic energy. This does not invalidate the “momentum of radiation” concept as presented by Poincaré. Planck’s notion of “electromagnetic energy elements” (18) had not as yet been presented. Using Planck’s notion of energy elements, one might conclude that if only such elements are considered, then the momentum of recoil is broken into something like “instantaneous” discrete momentum elements, where each would probably need to be the maximum momentum, $M_R = E/c$,

produced by a single energy element.

In 1900, the idea of the measure of mass was that it is “indicated” by “inertia,” which was the “quality” or “property” of matter. The result $m_r c^2 = E$ relates such radiant energy to an inertial mass. However, an inertial measure was also viewed as a “quantity” of matter as originally defined by Newton. This measure could not be altered by anything not itself considered as matter due to the principle of the “Indestructibility of Matter.” At that time, Poincaré believed that this m_r is there in the object but in a “disguised” form yet to be discovered. If one does not follow this 1900 prevailing notion, but follows the “absorbed” notion for the derivation and believes totally in the additive properties for measures of mass, then, in a non-recoil sense, it can be deduced that the object must have “converted” a certain amount, m_M , of its mass into m_r . (This, depending upon your viewpoint, is also inertial mass.) Such “proofs” as just given are, usually, stated as being “essentially” contained in the material used. Hence, for radiation, $E = m_M c^2$ is essentially a Poincaré result that would lead to a new interpretation for quanta of electromagnetic energy. Poincaré, at this time, does not go this far, for he considers m_r as a “fictitious” mass.

3. F. Hasenöhrle

In 1904, Hasenöhrle (10) specifically associates the mass notion via inertia with the energy concept. He investigates a system composed of a hollow enclosure filled with “heat” radiation and wants to determine the effect of pressure due to this radiation. His calculations lead him to conclude that “to the mechanical mass of our system must be added an apparent mass $mu = (8/3)E/c^2$,” where E is the energy of the radiation. (This is the first derived mass-energy “conversion” statement related to c^2 .) Later in a paper (11) [Received 26 Jan., presented 14 Mar.], he re-calculates this result and arrives at $mu = (4/3)E/c^2$. Hasenöhrle indicates that if the internal energy of a system consists of radiation, then, in general, the inertial mass of the system depends upon that energy. This is in accordance with his calculation. Thus, this new Hasenöhrle’s calculation establishes that due to the radiant energy, E , contained in his system, that to the inertial mass of his system must be added an apparent mass mu . The significance of Hasenöhrle results is not in the factor $4/3$ or any other such factor, but the concept that E is proportional to mc^2 . It’s claimed in 1914, Cummingham (1, p. 189), that Hasenöhrle made a slight error in that the shell is not included in his calculations. If the shell had been included, then the factor would be 1 or $mu = E/c^2$. (I have not been able to verify this claim. But, again this doesn’t matter since it is the concept that is the most significant part of Hasenöhrle’s contribution.)

4. A. Einstein

In the same journal where the Hasenöhrle article appeared and exactly two issues later, (4) [Received 27 Sept., presented 21 Nov.], Einstein presented a derivation using a two-observer approach and his Special Theory of Relativity that mass would be diminished by E/c^2 when it gives off radiation with energy E . This Einstein paper is an attempt to continue to popularize his Special Theory approach to such problems where the Special Theory, with a few applications, is presented in the previous issue (3) entitled “Zur Elektrodynamik bewegter Körper”. It’s interesting to note that the title of this second Einstein paper (4) is “Does the Inertia of a Body Depend upon its Energy Content” and this is answered, yes, by the Hasenöhrle paper, published in the same journal, that has a priority. Although the original “mass alteration” concept is implicit in Poincaré’s work and is specifically stated by Hasenöhrle, Einstein’s result, if correctly derived, would have reduced the Hasenöhrle factor of (4/3) to 1. It turns out, however, that Einstein does not give the correct derivation of $m_M = E/c^2$, with respect to radiation, for a body in but one reference frame.

Einstein, using the two-observer view, one on a platform in which the radiating body is stationary and one on a platform moving at a relative velocity v , claims that if radiant energy L is produced by an object, then the kinetic energy from this two observer point of view appears to change by (A) $K_0 - K_1 = L((1 - v^2/c^2)^{-1/2} - 1)$. He applies the usual procedure and expands the right hand side into a series. Then he gives a first order approximation statement, which is but the first term in this series. This is $K_0 - K_1 = (1/2)Lv^2/c^2$. Without any further explanation, he states that “if a body gives off the energy L in the form of radiation, its mass diminishes by L/c^2 .” Einstein apparently assumes that anyone reading this paper could make the substitution of the kinetic energy for a body moving at velocity v on the left side and divide, getting the difference in mass $m_M = L/c^2$. Of course, this is only an approximation.

This is the actual result using his derivation of (A). The standard kinetic energy expression is substituted on the left hand side and the $(1/2)v^2$ is divided. This is now a real change, not an observation only, for any single frame of reference. That is **if** the derivation is otherwise correct, then $E = mc^2$ for the radiation “given off” is only suggested and not derived. Indeed, there is a fundamental problem with this derivation. It mixes stationary and moving energy observations without taking into account the required relativistic alterations. Thus, this derivation does not actually say anything concrete about what the complete energy change would be as viewed from just one platform. Indeed, actual analysis by Ives shows that this result is forced to occur from some sort of prior knowledge and it

has not been derive (13).

Considering the Ives' claim, write down all of the known, at the time prior to the second paper, altered velocity “ v ” observations for the total energy of the body. These include the radiant energy L , the mass and the momentum of the body. Trivially, by combining the actual energy statements used in this derivation as observed from the moving platform for the body before and after emission of the radiation value L , where L is the radiant energy as observed from the platform where the body is stationary, the following is obtained. F_0 is the difference in the **two observed** energies of the body before emission. F_1 is the difference of the **two observed** energies after emission of L . But, K_0 is the kinetic energy from only one observer, the one on the moving platform before emission and K_1 is the kinetic energy again as obtained by the same observer on the moving platform after emission. Using these observations, it follows immediately that (B) $F_0 - F_1 = (L/((m - m')c^2))(K_0 - K_1)$, where m is the mass for the stationary observer before emission and m' the mass measured by the stationary observer after emission of the L . This is obtained prior to equation (A), a point prior to the approximating process.

Thus, from unit analysis, all that one can say using a correct Special Theory argument is that $L/(m_M c^2) = D = \text{some constant}$. However, an “It is clear” statement is made. The statement is that it is clear that $F_0 = K_0 + C$ and $F_1 = K_1 + C$, for some arbitrary C . And this is followed by (*) $F_0 - F_1 = K_0 - K_1$. However, this “it is clear” implication (*) only follows if $D = 1$. Hence, these statements force $L/(m_M c^2) = 1$, which is what is to be derive. This is followed in the derivation by the approximation process.

Indeed, it is this very “it is clear” statement that is objected to as not clear at all. This is seen in the actual expressions for F_0 and F_1 . $F_0 = H_0 - E_0$, and $F_1 = H_1 - E_1$. But this means that $H_0 = E_0 + K_0 + C$ and $H_1 = E_1 + K_1 + C$. Well, what’s wrong with this? The observations H_0 (the total energy), K_0 , H_1 (altered total energy), and K_1 are taken by the observer on the moving platform, the E_0 (total energy) and E_1 (totally energy after L is removed) are the total energy statements for the observer on the stationary platform. They need to be related in the usual relativistic manner not by the additive constant C . This manner will yield (B) not (A). Of course, if (*) can be fully justified, then no approximation needs to be made, since then $D = 1$ yields the absolute result.

5. M. Planck

The above Einstein approximation process did not go unnoticed. Max Planck made mention of it and I will quote Planck in a moment. Planck made an in-depth investigation of the energy “confined” within a body (19). Planck does not use the Einstein approach. Planck derives at the expression $m_M = E/c^2$ and states that “through every absorption

or emission of heat the inertial mass of a body alters, and the increment of mass is always equal to the quantity of heat divided by the square of the velocity of light in vacuo.” Then in a footnote on page 566 Planck writes, “Einstein has already drawn essentially the same conclusion [(4)] by application of the relativity principle to a special radiation process, however under the assumption permissible only as a first approximation, that the total energy is composed additively of its kinetic energy and its energy referred to a system with which it is at rest.” This approximation is all that Einstein achieved even with the error that $H = E + K + C$. Einstein’s statement of $E = m_M c^2$ is invalid since Einstein mixed two distinct observations without the required relativistic alterations to arrive at what is but an approximation. This approximation is the customary approach using the relativistic alteration in rest mass. But, does it have meaning for radiation? Assuming that Poincaré’s results are considered slightly incomplete (recoil replacing the idea of an inertial energy change), then the first complete derivation for one frame of reference is given by Planck as inspired by Poincaré. Of course, Planck’s result needs expansion by arguing that his result suggests a universal law.

The facts are, that I have never seen a rigorous single and exact proof that $E = mc^2$ is a **universal law** for all possible scenarios. However, Einstein’s 1905 proof is in error and this is not due to the fact that a first ordered approximation is used. Since partial derivations for $E = mc^2$ are only suggestive, then the little Poincaré derivation above for electromagnetic radiation (suggested 5 years prior to the Einstein’s radiation suggestion), the work of Hasenöhr and Planck’s are just as suggestive, if not more suggestive than the relativistic proof that, in general, $E = mc^2$ for all appropriate interactions.

In 1906 (5), Einstein applies his assumed correct radiation approximation that $E = mc^2$ and does reference Poincaré’s paper (21), entitled “La Théorie de Lorentz et le principe de la réaction,” and does give Poincaré credit for the mass-energy equivalence, at the least, for electromagnetic radiation.

In the present paper I want to show that the above theorem [$E = mc^2$] is the necessary and sufficient condition for the law of the conservation of motion of the center of gravity to be valid (at least in first approximation) also for systems in which not only mechanical, but also electromagnetic processes take place. Although the simple formal considerations that have to be carried out to prove this statement are in the main already contained in a work by H. Poincaré, [the previous mentioned paper] for the sake of clarity I shall not base myself upon that work (5, p. 252)

But, even with Planck’s complete derivation and this Poincaré acknowledgment, Ein-

stein later refused to accept any other priority for this notion. The title of his 1907 paper (6) appears to indicate why. It's entitled "On the Inertia of Energy Required by the Relativity Principle." In a paper written by Stark (24), Stark states that Planck gave the first derivation for $E = mc^2$. Einstein writes to Stark in 1908.

I was rather disturbed that you do not acknowledge my priority with regard to the connection between inertial mass and energy. (17 FEB 1908) (7)

Although his radiation derivation of 1905 is incorrect, he still considers himself as having priority over all of those that suggested an inertial mass-energy equivalence because, from his viewpoint, this equivalence could only be the result of "his" relativity principle. In his 1907 paper, Einstein states that it is not yet possible to generalize $E = mc^2$. This is a contradiction of the previous stance he took in his 1905 paper.

. . . we are led to the general conclusion: The mass of a body is a measure of its energy content. . . (4).

In his 1907 paper, referenced above, he considers two other special cases, where he obtains the mass-energy equivalence, again only in approximation, using the principles of relativity. Such derivations require some sort of "motion" of a "mass" to take place. One often uses the Special Theory mass-speed alteration statement. However, using a modern approach, the mass alteration equation is established by considering but one body and the Hamilton-Jacobi differential equation (12). For these cases, by defining the "relativistic kinetic energy," assuming the calculus models this "energy," and by arbitrary choice the approximation is eliminated (25, p. 70). The derivation in this reference is only partially correct in that the c is assumed not to vary.) The approximation is usual obtained by expanding $mc^2 = m_0c^2/(1 - v^2/c^2)^{1/2}$ into an infinite series, where m_0 is the "rest" mass. This gives $mc^2 = m_0c^2 + (1/2)m_0v^2 + (3/8)m_0v^4/c^2 + \dots$. This series contains the m_0c^2 term and other "energy terms" such as a term for classical kinetic energy. In this case, the approximation is made exact by claiming that all terms, other than m_0c^2 , represent an additional kinetic energy effect. Then one "selects," rather than derives, $E = mc^2$ and $E_0 = m_0c^2$ apparently based upon the previous incorrect Einstein radiation "proof."

As I'll discuss below, the Einstein "priority" quotation is a very unfortunate stance to take relative to priority and the furthering of science. Unfortunately, as shown in 1994 (12), Einstein made a logical error when he extended his Special Theory results. The new derivation in (12) shows that there is a cause for this mass-energy equivalence in that all such relativistic behavior is caused by an electromagnetic interaction with a "substratum." This shows that such radiation derivations as those of Poincaré, Hasenöhl and Planck are

in complete accord with any of the special case “relativity principle” derivations, when correctly interpreted.

The conjectured universality of the $E = mc^2$ is not the reason for this article. This article is written to demonstrate priority for such a radiation notion and possible implications. Consequentially, I consider Poincaré as presenting the first known correspondence between the energy of radiation, in general, and a mass concept - “momentum.” This essentially leads to $E = mc^2$ for radiation. Then Hasenöhrl should also be given credit for the discovery that a given radiant energy E is directly proportional to an associated apparent mass of approximately the value E/c^2 ; and if the energy is internal to the object, then this radiation increases the inertial mass. In my view, Hasenöhrl should also be given credit, by obvious implication, for the discovery that the “absorption and emission” of such radiation increases or decreases, respectively, inertial mass. Finally, Planck, using the Poincaré notions, should be given credit for his derivation that E increases or decreases the inertial mass of a body by the amount E/c^2 . I can find no mention of Hasenöhrl in any Einstein paper on this subject from 1900 - 1909. It’s hard for me to believe that Einstein did not know of the Hasenöhrl paper that appears in the very journal in which Einstein later published his original findings. In order to achieve the same result, did Einstein use Hasenöhrl’s suggested mass-energy correspondence as motivation for his faulty relativity principle derivation?

6. H. Poincaré Again

Today, Poincaré is not given the basic credit for the idea that electromagnetic energy has a “mass” component and the immediate derivation, above, from his work that the electromagnetic energy radiated $= m_M c^2$. We all know that Poincaré is sometimes given priority for the Principle of Relativity which he states in his talk “L’etat actuel et l’avenir de la physique mathematique,” at the Congress of Arts and Sciences, St. Louis, Sept 24, 1904 and where he identifies the Lorentz transformation group properties (22). Now to a more startling notion of Poincaré’s that is employed in much research today. Einstein’s first Special Theory paper arrived at Ann. Physik on 30 June 1905. But on 23 July 1905 at the editor for Rendiconti de Circolo Mathematico di Palermo a paper by Poincaré of the title “Sur la dynamique de l’electron” arrived. In this paper, Poincaré applied the contraction notion from the Lorentz transformations to electromagnetic theory and **to gravitation**. “Poincaré published in 1905 a note (followed by an extended memoir) on the dynamics of the electron, containing the whole mathematics of special relativity” (14). Poincaré makes the following remarkable observation.

If we accept the postulate of relativity [his postulate], we shall find that

among the laws of gravitation and the laws of electromagnetics there exists a common number. It is the velocity of light. We shall find that it occurs in all forces, of whatever origin, and it can only be explained in two ways: (1) Either there exists nothing in the universe that is not of electromagnetic origin; (2) or, this quantity, which is common to all physical phenomena, appears only because it relates to our methods of measurement.

The universal law as seemingly postulated by Einstein from such evidence as the radiation energy-mass correspondence does not give an actual causal relation. But Poincaré's statement (1), as far as a causal statement is concerned, may be totally correct. There is evidence that mass and other effects might actually be caused by something similar to the postulated "Zero-point electromagnetic field" (See 9, 23, and The Theory of Infinitesimal Light-Clocks (12) where such a correspondence is predicted.) If this is accepted as the case, then the radiant energy and its correspondence to a mass measure takes on additional significance.

Why might it be that Poincaré is not given the significant credit for many of these basic ideas? Poincaré has two strikes against him. He is French, a mathematician first and a physicist second. The physics community uses, although often not very rigorously, the tools created by the mathematician. However, they often have difficulty with accepting a mathematician's work when the mathematician steps into the "their" discipline. Then, as is now well know, Einstein is classified as a rather marginal mathematician when compared to Poincaré and others (15). Further, there are other possible reasons that deal with the politics of Poincaré's time (14). Since it has been established how the "essentially" derived, approximately derived, or exactly derived statement $E = m_M c^2$ by Poincaré, Hasenöhl, and Planck are related to the originally published Einstein approximation, should this lack of recognition for these scientists continue?

7. Final $E = mc^2$ Remarks

Poincaré, Hasenöhl, and Planck are not the only individuals that have a certain priority relative to $E = m_M c^2$. According to Professor Umberto Bartocci, Olinto De Pretto published the expression $E = mc^2$ in the science magazine (2) in 1903. His expression is a speculation that is not derived from more fundamental principles. There is evidence that Einstein was aware of the De Pretto speculation and that this is an additional driving force behind his attempt to derive this expression for radiation, at the least. There is also very strong evidence that Einstein never gave De Pretto any credit for his great insight. It is an absolute requirement that one must do a certain amount of literature "research" prior to publishing a claimed new disclosure. This is done to determine if, indeed, your

claimed disclosure is new, or to give credit to others that have certain levels of priority if your derivation is obtained by other means. There is no doubt in my mind that Einstein would have known of the last Hasenöhrl paper since it appeared in the principle journal that Einstein used six months later to publish his own (1905) derivation. If I am correct, then Einstein would thus have been aware of Hasenöhrl's first paper as well. Poincaré was a very well known mathematician and scientist who won the first Bolyai prize, a prize that Einstein did not win when nominated by Hilbert. I do not speculate any further as to why, today, proper credit is not being given to the contributions of Hasenöhrl, Poincaré, Planck and De Pretto.

8. Einstein's God

It is certainly proper that this article conclude with some of Einstein's deity beliefs. Whatever attributes Einstein's god may have, they are not those expressed by a basic interpretation of the Old Testament. In his 1946 autobiographical notes, he writes about his childhood, "Through the reading of popular scientific books I soon reached the conviction that much in the stories of the Bible could not be true. The consequence was a fanatic freethinking coupled with the impression that youth is intentionally being deceived by the state through lies; it was a crushing impression. Mistrust of every kind of authority grew out of this experience, a skeptical attitude toward the convictions that were alive in any specific social environment - an attitude that has never again left me, even though, later on, it has been tempered by a better insight into the causal connections" (8).

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