Solutions to the "General Grand Unification Problem," and the Questions "How Did Our Universe Come Into Being?" and "Of What is Empty Space Composed?"*

Robert A. Herrmann

Mathematics Department
U. S. Naval Academy
572 Holloway Rd.
Annapolis, MD 21402-5002

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Abstract: Using mathematical techniques to model one of the most simplistic of human linguistic processes, it is rationally predicted that within the nonstandard physical world there exists a force-like (logical) operator ${}^*\mathbf{S}$ and an entity w' such that ${}^*\mathbf{S}\{w'\}$ sequentially generates each of the Natural (i.e. physical) systems that comprise a Universe. This scientific model shows specifically that within the nonstandard physical world the behavior of each Natural world Natural-system is related logically. Further, the model predicts the rational existence of a single type of entity within the nonstandard physical world's substratum that can be used to construct, by means of an exceptionally simple process, all of the fundamental Natural (i.e. physial) world particles used within particle physics. In important section 11.2, it is shown how (Natural (i.e. physical) law) allowable perturbations in Natural-system behavior are also included within this mathematical model. These results solve the pre-geometry problem of Wheeler. In general, the model predicts that when the behavior of these Universe creating processes are viewed globally, they can be interpreted as the behavior associated with a powerful intelligent agent. (An extensive refinement has been added to the original and further refinements are find in the (2014, 2013) references.)

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*rah@usna.edu or drrahgid@hotmail.com

1. History.

[Note: References for this section appear on pages 4 – 5.] In August 1979, I was to give a paper before a gathering of mathematicians at the Summer Meeting of The American Mathematical Society [1]. This summer meeting is a joint meeting with the Mathematical Association of America and was being held at the University of Minnesota at Duluth. I boarded an airplane at Chicago for the last leg of my trip, and who should be setting next to me at the window seat but John Wheeler, the Joseph Henry Professor of Physics emeritus at Princeton. Professor Wheeler was to deliver an invited talk before the Mathematical Association of America on what he believed was a fundamental difficulty with the philosophy of science associated with physical theories.

We discussed various things as we passed over some magnificent Minnesota thunderheads. Looking out of the window Professor Wheeler made the curious but simple statement, "We can't do that." What does this statement mean and who are the "we"?

After more discussion, the meaning of his comment was clear. First, the "we" are physicists and what they can't do was illustrated by the thunderhead. The thunderhead is assumed to be a collection of Natural (i.e. physical) systems and, as a collection, can also display emerging properties. You have the water droplets, or crystals. Then their paths of motion caused by internal forces. The various electric potential differences produced by such motion and hundreds of other factors that scientists claim contribute to the overall behavior we observed from our window.

A Natural-system is a set or arrangement of physical entities that are so related or connected as to form an identifiable whole. Many Natural-systems have an associated physical theory. These theories are used to predict behavior for each Natural-system. BUT many of these theories use different methods to predict behavior. Indeed, one often has to restrict a particular theory to specific areas of application. When this is done you often have a boundary between two theories – an overlapping region – where one or the other but not both apply for the methods used in one of the theories are inconsistent with the methods used in the other theory. The hundreds of Natural-systems (now called subnatural-systems) that comprise the Natural system called a thunderhead are then put together in some manner to obtain the entire thunderhead one observes. With respect to the universe as a whole, the differences between various predicting theories are profound. What Professor Wheeler meant by the ". . . can't do that" with respect to our universe is the acknowledgment that physicists had not found a general unifying theory that predicts all of the behavior of all subnatural systems that comprise the entire Natural-system called our universe. Science needed a theory that would be totally consistent, and a theory that would replace the piecemeal approach. Of course, such a unifying theory did not even exist for the behavior of the thunderhead we observed.

I restated this problem in the following manner.

> **Does Nature really combine subnatural-systems together, in a way that we can understand, to produce an entire Natural-system or does it use an entirely different method so that inconsistencies are somehow avoided? Is there something else required, something more basic than science has yet described and that's needed to combine all the Natural-systems together that comprise our universe? Indeed, how can a universe that's perceived by humans to have order and harmony really be a product of chaos?**

By the way, I added that last question to this problem since it is certainly relevant.

After our discussion, it occurred to me that something discovered in October 1978, and now called ultralogics [2], might possibly lead to an answer to these questions. But I also knew that the answer might be rather startling.

The standard approach is a piecemeal "bottom-up" approach beginning with the "bottom," so to speak, of the scientific hierarchy (or "domain of explanation"). Supposedly, if you can find a unification of the fundamental forces (or interactions), then this would lead to an "upward" process that would eventually unify all physical theories. As discussed above, this is a doubtful assumption. Moreover, many scientists and philosophers of science believe that there are emergent properties of organisms that cannot be fully understood as products of DNA or chemistry and, thus, do not follow directly from the four fundamental forces.

On the other hand, there may be a *top-down* approach that answers the Wheeler questions. This means to find some scientific theory that gives processes which yield cosmologies, cosmologies that contain all of the processes that control the behavior of all Natural-systems.

The "deductive-world model," (i.e. the D-world model) was the first constructed (1978) and is interpreted in a linguistic sense. D-world model properties will not be discussed directly within the following pages. Many of these properties can be found in sections 1 – 5 of the book *Ultralogics and More* [2]. However, some of what appears in the next sections does have the requisite linguistic interpretation associated with the D-world model. The actual model used to solve these problems is generally termed the *General Grand Unification Model (GGU-model).* (For a special purpose, a portion of the GGU-model, the *metamorphic-anamorphosis model* (i.e. the MA-model) is employed.) Further, the intuitive concepts and mathematical methods that determine the GGU-model properties are very similar to those used for the additional D-world model conclusions. Thus becoming familiar with the GGU-model conclusions will aid in ones comprehension of the D-world model.

The basic construction of the GGU-model uses the following philosophical stance as described by deBroglie.

> *. . . the structure of the material universe has something in common with the laws that govern the workings of the human mind* [3, p. 143]

It was determined that an appropriate cosmogony would depend upon the properties of a logic-like operator and subsidiary concepts associated with the *nonstandard physical world* (i.e. NSP-world) [4, part III]. The construction of the GGU-model began in August 1979 and a series of announcements [5 – 10] relative to its mathematical construction appeared in the Abstracts of papers presented to the American Mathematical Society. In particular, [5] mentions the logic-like operators.

In 1982, papers by Bastin [11], and Wheeler and Patton [12] were discovered in *The Encyclopaedia of Ignorance.* I mention that [12] does not contain an important appendix that appears in the original 1975 paper. The paper by Bastin describes the *discreteness paradox* and the paper [13], using a few GGU-model procedures, presents a solution to this paradox. Next the requirements for an acceptable cosmogony stated in [12] were compared with GGU-model properties. A major difficulty in showing that the GGU-model meets all of these requirements is in the language used to give physical-like meaning to the abstract entities. After considerable reflection, the prefix "ultra" was decided upon. The use of this prefix is consistent with its use in the construction of the nonstandard structure. A relatively explicit construction of such a structure uses the concepts of the ultrafilter, the ultraproduct and the ultralimit. In 1987, a paper justifying the fundamental mathematical theory of nonstandard consequence operators was published [14].

Although a few technical aspects associated with the GGU-model were yet to be fully justified, two papers were written in 1986 [15], [16] announcing the solution to the general grand unification problem and the pregeometry problem of Wheeler and Patton. To properly prepare paper [16], the original Wheeler and Patton paper [17] was utilized. An appendix in [17] describes how Wheeler and his colleagues at Princeton tried to construct a pregeometry from the statistics of very long propositions and very many propositions, where the term "proposition" refers to the propositional (i.e. the sentence) logical calculus. They failed to achieve a solution, but Wheeler left open the possibility that a solution could be obtained using concepts from the area of Mathematical Logic. This is exactly what has occurred.

Beginning in about 1989, a project was instituted to justify fully all of the GGU-model concepts, among other aspects of the NSP-world, and to present them in monograph form. The basic monograph [2] was completed in 1991 and the final result that completes the justification process was published in 1993 [18].

In the next section is given a very brief, general and mostly nonmathematical description of how the GGU-model and the language of the NSP-world solve the problems mentioned in the title. Presented in the third and technical section titled "The Mathematics," with additional comments, is most of the actual mathematics that, when interpreted, describes the properties of the GGU-model [2] that correspond to the nonmathematical descriptions.

The following quotations are taken from [17] and are all relative to the Patton and Wheeler cosmogony requirements.

(1) *Five bits of evidence argue that geometry is as far from giving an understanding of space as elasticity is from giving a understanding of a solid.* [17, p. 539]

(2) *They also suggest that the basic structure is something deeper than geometry, that underlies both geometry and particles ("pregeometry").* [17, p. 539]

(3) *For someday revealing this structure no perspective seems more promising than the view that it must provide the universe with a way to come into being.* [17, p. 539]

(4) *It brings us into closer confrontation than ever with the greatest questions on the book of physics: How did our universe come into being? And of what is it made?* [17, p. 540]

(5) *Tied to the paradox of the big bang and collapse is the question, what is the* <u>substance</u> *out of which the universe is made?* [17, p. 543]

(6) *But is it really imaginable that this deeper structure of physics should govern how the universe came into being? Is it not more reasonable to believe the converse, that the requirement that the universe should come into being governs the structure of physics?* [17, p. 558]

(7) *It is difficult to avoid the impression that every law of physics is "mutable" under conditions sufficiently extreme, ...* [17, p. 568]

(8) *It is difficult to believe that we can uncover this pregeometry except as we come to understand at the same time the necessity of the quantum principle, with its "observer-participator," in the construction of the world.* [17, p. 575

(9) *. . . . a guiding principle, is what we seek.* [17, p. 575]

As the GGU-model pregeometry is discussed, I will refer to these quotations where applicable. Relative to (9), the first half of the guiding principle is the deBroglie statement viewed from the NSP-world. The second half of the guiding principle is relative to the philosophy of realism and an observation made by the originator of nonstandard analysis Abraham Robinson. First recall that Newton believed that infinitesimal measures were real measures associated with objectively real entities. Berkeley and Leibniz did not accept Newton's belief. Robinson at the end of his very first published paper on nonstandard analysis made the following statement relative to the modern concepts of how mathematical models are used to predict indirectly Natural-system behavior.

> *For phenomena on a different scale, such as are considered in Modern Physics, the dimensions of a particular body or process may not be observable directly. Accordingly, the question whether or not the scale of non-standard analysis is appropriate to the physical world really amounts to asking whether or not such a system provides a better explanation of certain observable phenomena than the standard system. . . . The possibility that this is the case should be borne in mind.*

Fine Hall,
Princeton University [19, p. 440]

Robinson is referring to infinitesimal measures in this quotation. Since the publication of [19], nonstandard analysis has been applied to entities that are not infinitesimal in character. The second half of the guiding principle is an extension of this Robinson statement to the NSP-world. Thus the acceptance of the NSP-world as a viable realism depends upon whether or not it provides a "better" rational explanation of certain observable phenomena than the standard world model. The philosophy of what constitutes "a better explanation" is left to individual choice.

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Important Reference

The above reference [2], from which the mathematical portions of the monograph are taken, contains **all** of the fundamental concepts associated with the methods and processes used within the discipline of nonstandard analysis as they would apply to all aspects of the GGU-model and D-world model.

2. Two General Discussions

[Note: This first discussion is a portion of a general audience non-technical and elementary talk I gave on this subject. The numbers that appear in the double square brackets refer to the quotations from [17] as they appear on pages 5.] Let me point out that as long as scientists use mathematics to obtain their theories and written symbols, diagrams, photographs and the like to communicate their concepts then the GGU-model can't be eliminated. It will always be there lurking in the background.

Now to answer the question "How was our universe created?" Let's start by considering a single geometric point a few feet in front of you. A geometric point in this sense is a *position* in our universe and, for the present, has no other meaning. Now, I'm able to magnify this point for you by using a mathematical microscope with a power that's greater than any power that can ever be achieved by human means.

Suddenly, you see the point open up, like the iris of your eyes. What's revealed to you is a *background universe*, a *substratum*, or whatever you might like to call it. Now, the Natural-world is the world we can scientifically perceive, and this Natural-world point is still in your field of view with a small portion of the background universe surrounding it. You can't make out much detail, but there's definitely something there. The detail you see is sharper and clearer near to the single Natural-world point. Then clarity slowly fads as you proceed further from that one solitary Natural-world position within our universe. You can find no clear outer edge within your field of view. This background universe forms a portion of what I called the nonstandard physical world - the NSP-world ([4] above). [Note: The entire collection of all possible standard world points coupled with all of the NSP-world points that are infinitesimally near to them is called the *finite* or *bounded* portion of the NSP-world.] The term the NSP-world is also used for other applications. However, for our purposes I'll discuss a portion of this NSP-world, the GGU-model.

Our universe is inside and "just as near to" the NSP-world as I have described it. One might say that this background universe is scientifically omnipresent. Now the GGU-model portion of this background universe specifically states that there will *never* be a human language that can give a completely detailed description for the mechanisms that may have produced our universe, but there do exist such mechanisms within this background universe. Such mechanisms exist but, no matter how hard we try, the human mind can't comprehend *all* of the *details*. Well then, what can be known about how universes, such as ours, can be created by NSP-world mechanisms? We can know general properties.

To begin with, there exists within the NSP-world a set of "things." These "things" need not be considered as being within our Natural universe. But why do I call them "things"? I do know a lot about these "things." But, unfortunately, if I were to give you a basic description for their contents, I would only use various mathematical terms from the fields of mathematical logic and set-theory. The most difficult task I've faced is to give some "physical" meaning to these "things." These objects could easily fall into the category of those objects within the NSP-world that have NO human language physical descriptions at all. I know that these "things" behave like informational "superballs."

After literally years of reflection, it was determined that these "things" can be described as containing all of the building plans, the laws of "Nature" and even step-by-step images of how an ideal universe will appear as it develops. They also contain a great deal of information that is incomprehensible to the human mind. It's interesting to note that, after I came to these conclusions, I came across a quotation from one of the greatest scientists of our time. Hermann Weyl is quoted as saying the following:

> *Is it conceivable that immaterial factors having the nature of images, ideas, 'building plans' also intervene in the evolution of the world as a whole?*

We have "things" in the background universe that are ready, when the conditions are just right, to *aid* in the production of universes, ours included. And different "things" aid in the production of different universes. Actually, these "things," which I have called *ultimate ultrawords*, (or now *ultra-logic-systems*) don't work alone. Each is just one piece of an entire process. All pieces of the "puzzle" must be put together before a universe is produced. Now the existence of these ultra-objects may not seem startling, but a lot more is yet to come.

What are the conditions that must exist in the NSP-world – conditions needed to trigger the creation of a universe such as ours? At present, there is no human scientific language that can detail these conditions, no human understanding of what these conditions are or were, only that such conditions exist.

I remind you that this cosmogony comes from a mathematical model, a model that can't be eliminated from modern science. This cosmogony satisfies all of the basic theoretical requirements of the scientific method. The universe in which we dwell, our solar system, the Earth, or a virus are Natural-systems. (Now called physical-systems.) Natural-systems are studied by the physical scientist in a piecemeal fashion. They apply distinct procedures that seem to describe the moment-to- moment behavior of each distinct Natural-system. Now within the NSP-world there's <u>one</u> special process called " ***S**," it's a hidden process, an *intrinsic* (hidden) *ultranatural process* (IUN-process).

The ***S**-process is one of the entities know as an *ultralogic.* When the conditions are just right, this force-like process takes one of these ultimate ultrawords and produces a universe. Indeed, it combines together, controls and coordinates all of the distinctly different Natural-systems that comprise a universe. The ***S**-process applied to an ultimate ultraword produces each Natural-system and also yields the moment-to-moment alterations in the behavior of each and every Natural-system ⟦2, 3, 4, 6⟧. But, how does it do this? We can't know many details, but a few simple mechanisms do present themselves.

Within the NSP-world there are objects, of a single type, (that I term *ultimate subparticles*) that, from the NSP-world viewpoint, can be "easily" combined together to produce every material object, electrons, protons, our earth, and everything else that might be termed material as well as immaterial fields, if such exist ⟦2, 4, 5, 6⟧. This combining process cannot be reproduced by Natural means within any laboratory within our universe. (Note: 12/12/12 Subparticles are now called "propertons" and ultimate subparticles are called ultra-propertons.)

What about the development of our universe? That is how it changes with respect to time. This force-like process does produce a "beginning" for our universe and, as mentioned, in a remarkable step-by-step manner, it produces in the proper "time" ordered sequence all of the material changes from the very beginning until the universe arrives at a stage such as that which we observe about us. It produces all the Natural events that constitute the moment-to-moment changes that alter the appearance of a universe. This remarkable force-like process applied to an ultimate ultraword ((12/12/12) an ultra-logic-system) yields a solution to the general grand unification problem.

I say that this process is "remarkable" but I haven't explained why. I give you one example. From human perception, we often characterize certain Natural changes in a Natural-system as chaotic or random. This means that there seems to be no pattern for such changes – that is no pattern that can be comprehended by the human mind. Indeed, no human predictions can be made as to how individual objects will behave from one moment to another. From our viewpoint, there are no harmonious or regular laws that can produce such individual changes. It can be shown that from the NSP-world viewpoint the opposite is true. This seemingly irregular behavior within our universe is actually only what **we** can perceive of what is, in reality, an extremely regular process. How is this possible?

Well, it turns out that as a universe develops, as it changes, there are millions of other events taking place within the NSP-world that we can't perceive scientifically. These *ultranatural events* sustain and hold our universe together, so to speak. It's the ultranatural events that are combined together with the Natural events we perceive that actually comprise a complete change. Thus it's simply a matter of perception. I wish there were words in any language that fully describe this wondrous "combining" together process. There is a technical term, "ultrauniform," that can be used. But this gives no indication of what is actually occurring. The steps in this combining together process are so minuscule, so small, so refined that the human mind can't fully appreciate nor comprehend the ***S**-process. It doesn't correspond to anything that can at present be perceived or imagined by us.

Another startling aspect of the ***S**-process is that every Natural event is connected, within the NSP-world, with every other Natural event. No Natural changes, from the NSP-world viewpoint, are independent from one another. A Natural event taking place in one galaxy is related by the ***S**-process, to events taking place in every other galaxy. But, unfortunately, we can't know, except in general terms, the actual composition of an ultimate ultraword nor describe in any human language the necessary ultranatural events that sustain this process.

Now, what I've described, as best as I can, is the creation of an "ideal" universe. But what happens when there exist creatures within a universe that can alter its ideal development? Well, these creatures can only alter the Natural events, they can't alter the ultranatural events. No matter what these creatures do this "glue" that holds the universe together still remains ⟦8⟧. I repeat, we can have no knowledge as to what these ultranatural events are. They cannot be described in a human language, ever. All we know is that they exist.

From the human perspective, there can be "sudden changes" in the behavior of a Natural-system at any time during its development ⟦7⟧. But these changes are not truly "sudden" from the NSP-world viewpoint. Indeed, we can magnify a point in time, as we did with a point in space, and investigate what happens at the moment of change. Again wondrous events occur that are difficult to describe in a human language. Technically, the changes occur in an *ultrauniform* manner. This type of change, from the NSP-world viewpoint, can be compared with the mathematical concept of an "uniformly continuous" change. But ultrauniform is "infinitely" more uniform, so to speak, than the usual standard concept of an uniformly continuous change.

. . . .

If we look at the GGU-model as a whole, is there a way to describe it in its entirety? Yes, there is and this may be the most remarkable aspect of this research. The GGU-model can be characterized as behaving in its entirety like a super, super, super to an infinite degree, *mind* or possibly a *computer*. The processes are similar to how an almost inconceivably powerful mind or maybe a computer would behave.

In the next section titled "The Mathematics," is reproduced, with additional comments, some of the actual mathematics that, when interpreted, describes the properties of the GGU-model that correspond to the nonmathematical descriptions. [The procedures in this section have been highly refined and added to. The notion of "instructions" and "instruction-information" are now the fundamental entities that generate a universe. (See appendix.)]

This portion of this discussion will be somewhat more technical than the previous portion. I will refer to section 3 as the motivation behind this mathematical model is discussed. The expression "Natural world" refers to the collection of all entities that are categorized as Natural-systems.

Relative to the behavior of a Natural-system, a general scientific approach is taken and it is assumed that scientists are interested in various types of descriptions for Natural-system behavior. It is not difficult to show that all forms of scientific description can be reduced to strings of symbols. Developmental paradigms are simply time related descriptions for the ideal behavior of any Natural-system viewed simply as strings of symbols. Although some of these collections of descriptions might be generated by a specific theory, a general approach that a developmental paradigm describes a sequence of Natural events is taken. The expression *Natural event* means an objectively real and physical occurrence that is categorized as "natural" by the physical scientist. This approach does not include any requirement that such a sequence be generated by some accepted and humanly comprehensible theory. On the other hand, theory generated sequences are not excluded.

On the first six pages of the next section are reproductions of pages taken from the paper "Nonstandard consequence operators" ⟦Herrmann, R. A. *Kobe J. Math.* 4(1987): 1–14⟧. They detail how the basic somewhat unusual model $\mathcal{E}$ is constructed. The terminology used is that of the abstract mathematical structure. For example, certain subtle consequence operators are interpreted as ultralogics, while an ultraword is an unreadable sentence. The theorems on these pages show some of the behavior of nonstandard consequence operators. The most significant discussion on these first six pages is the construction and embedding of the set $\mathcal{E}$. Although the results on these first six pages show some of the interesting behavior of nonstandard consequence operators they are not specifically needed to comprehend what follows. Further, the "time ordering" concept considered throughout what follows can be replaced with the notion of the "universal event number."

Since an ultralogic is based upon the selection of some nontrivial logical process, there is a need to select a logical system that is common to all known logical systems used throughout scientific discourse. After some difficulty, a system, denoted by **S** and discussed at the beginning of section 7.3 titled ultrawords, was selected. I point out that the ordering of the sections of the included mathematics is not the same ordering in which the processes were originally discovered and used. Intuitively, throughout the modeling of these linguistic concepts, a specific "frozen segment" is considered as a description for a particular Natural event that occurs at a particular moment of time. From a scientific communication point of view, this description is all that can be scientifically known about such an event and is substituted for it.

Developmental paradigms, the deductive logic **S** and the like are embedded within a special but well-know mathematical structure called a superstructure by means of a fixed encoding and are further embedded into a nonstandard structure. The next step in the modeling process is to show that for a developmental paradigm written in a standard language there exists a new object, called an ultraword, that when the ultralogic (an intrinsic ultranatural process (i.e. an IUN-process)) $^*\mathbf{S}$ is applied to this ultraword the entire developmental paradigm (or the corresponding Natural event sequence) is logically produced. Theorem 7.3.1 establishes that for each developmental paradigm such an ultraword exists. Defining the *Natural world* as the collection of all Natural event sequences that correspond to the behavior of all systems that are categorized as Natural-systems, it follows that ultrawords cannot be entities within the Natural world under the GGU-model interpretation. Also the force-like operator $^*\mathbf{S}$ cannot be applied within the Natural world. Rather then simply accepting that ultrawords are "things" that cannot be further described, elsewhere additional intuitive meaning for this concept is discussed.

Relative to the collection of all events generated by the force-like process $^*\mathbf{S}$, a question that arises. Are only the developmental paradigm events obtained as a result of the process $^*\mathbf{S}$ applied to an ultraword w? A conjecture was no. Theorem 7.3.2 shows that other "descriptions" for other types of events also occur when $^*\mathbf{S}$ is applied to a specific w. A later investigation, Theorem 10.1.1, shows the general composition of these new descriptions or events. But more importantly, Theorems 7.3.2 and 10.1.1 show that these new events must occur. Further, they cannot occur within the Natural world and cannot be described by any natural language. These events have been interpreted as events (called ultranatural events (i.e. UN-events)) that are needed to uphold and sustain a Natural-system's development.

The above discussed results do not answered Wheeler's basic question. The basic question is answered by Theorem 7.3.4 in an slightly more general mode. Every subnatural-system within the Natural-system called the universe can be associated with its own special system generating ultraword w'_i. Corollary 7.3.4.1 says that if we select a set of ultrawords that generate each and every subnatural-system that is within our Natural universe, then there (logically) exists an ultimate ultraword w' such that when the force-like process ${}^*\mathbf{S}$ is applied to w' each of the subnatural-system ultrawords are produced. Hence, all of the original subnatural-systems (the Natural event sequences) are produced by application of ${}^*\mathbf{S}$ to w'. Thus within the nonstandard physical world, not within the Natural world, there (logically) exists a force-like process the combines all of the Natural-systems together in a consistent manner and yields our Natural universe and all of the moment-to-moment alterations that comprise its development. The mathematical existence of the ultimate ultraword w' yields a solution to the "general grand unification problem" as described by Wheeler.

Since the Natural event sequences are not necessarily predicted by a theory, then is it possible that the ideal developmental paradigms and the corresponding Natural event sequences are somehow "preselected"? Theorem 7.2.1 coupled with the discussion in 10.4 yields the necessary NSP-world IUN-selection processes. It is at this point that one of the most basic and significant properties of the GGU-model becomes apparent.

Suppose that you consider a partial denumerable developmental paradigm d_i where one of its members F_i is a frozen segment that represents a specific configuration for a Natural-system as it appears at a standard time interval $[t_i, t_{i+1})$ Notice that $\mathbf{F}_i \in \mathbf{T}_i$. The (cosmic or standard NSP-world) time t_i is conceived of as the moment of time in the past when a time fracture occurs and all other members of d_i represent behavior "after" such a time. There are infinitely many sequences (finite or denumerable) of standard frozen segments or *-frozen segments that can be adjoined to d_i, and that yield other developmental paradigms that describe Natural or ultranatural behavior for the same Natural-system but for cosmic times "prior to" t_i. This yields type d or type d' development paradigm.

Theorem 7.2.1 states that there is an external IUN-selection process that yields each of these developmental paradigms. A developmental paradigm that contains no additional frozen segments of any type prior to t_i along with its associated ultraword represents the maximum scenario. Note that a maximum scenario can yield by application of an ultralogic, *-frozen segments as well as frozen segments. A developmental paradigm that contains only time ordered standard frozen segments before and after t_i is a minimum scenario. An intermediate scenario contains some or all *-frozen segments representing ultranatural events prior to t_i. "Sudden alterations" are modeled by the minimum and intermediate scenarios and can be used for various purposes such as the Patton and Wheeler concept of "mutability" of Natural law or behavior. For the minimum and intermediate scenarios, can we investigate what happens during the NSP-world time when Natural law or Natural constants are altered? Can we "open up the time fracture," so to speak, and look inside?

As discussed in section 7.5, applications of Theorem 7.5.1 yield a startling view of how these alterations are being made within the NSP-world. Furthermore, the force-like process ${}^*\mathbf{S}$ that produces all of the event sequences does so in a remarkable manner as described by Corollary 7.4.1.2.

As to various NSP-world objects that can mediate any GGU-model alterations, produce informational transmissions, and be considered as the composition of the vacuum, these are automatically generated by this mathematical model as shown by Theorem 9.3.1 where the descriptions are interpreted as descriptions for objects.

As to how the assumed "Laws of Nature" that seem to exist today came into being, the discussion in section 10.2 shows that these can also be assumed to have been produced by ${}^*\mathbf{S}$ applied to an ultraword. This answers another basic Wheeler question. The GGU-model cosmogony yields the various Natural laws that exist today. Further, an ultraword such as w" gives an external unification to the collection of all written physical theories.

The last section in this paper deals with the "substance out of which the (Natural) universe is made." It shows that using the method outlined in Theorem 9.3.1 the basic properties of propertons can be obtained. *Ultra-propertons* are obtained with the aid of Theorem 11.1.1. Other properties of the infinitesimal and infinite hyperreal numbers coupled with a very simple hyperfinite translation (affine operation) lead to intermediate propertons that can be finitely combined together to produce every basic entity within our Natural universe.

[Note (1): Relative to cyclic, multi-universe, plasma or any cosmology that claims that our universe has no Natural time beginning or no Natural time ending, such universes still have a "beginning" from the GGU-model viewpoint. No result in this paper is dependent upon a universe existing for only a finite period of time. Each Natural-system still has a beginning and ending with respect to an identified cycle. There is for each Natural-system within an ith cycle an ultraword w_j^i. Then there is a cycle ultimate ultraword $(w^i)'$. The multi-cycle-universe generating ultimate ultraword w' still exists and the force-like operator ${}^*\mathbf{S}$ still applies. That is, that w' generates each cycle and the contents of each cycle. Also note that if a Natural-system j is open-ended in that it continues to alter its appearance for all of time and has either no beginning or no ending in the intuitive sense, then this also can be modeled by considering a denumerable sequence of basic time intervals $[a_i, b_i)$. Again there is an ultraword w_j^i for each $[a_i, b_i)$ and, hence, an ultraword $(w_j)'$ that generates each of the w_j^i. Thus $(w_j)'$ generates the entire Natural-system's behavior. Finally, there is an ultimate ultraword w' that generates each $(w_j)'$ and, hence, each Natural-system. 3 MAR 1996]

[Note (2): For an ultimate GGU-model conclusion that shows how an GGU-model generated universe can be made to vary due to moment-to-moment perturbations, see section 11.2 that starts on page 55. This ultimate GGU-model conclusion also specifically models the "quantum" participatory requirements. 24 DEC 1996]

[Note (3): As mentioned previously, the set of all ultranatural initial conditions that would lead to universe creation can be entirely compose of entities that behave like"initial conditions" but these conditions cannot be described in any language associated with any entity within our universe. This fact does not mean that this set does not contain some ultranatural initial conditions that can be properly described by means of a comprehensible language. One such description follows the pattern of "field fluctuations." The collection of all ultimate ideal ultrawords w' and its collection of perturbative ultimate ultrawords $\{w_t'\}$ within the NSP-world substratum can be supposed to form a dense collection W'. These collections fluctuate fortuitously with respect to NSP-world spacetime. Whenever a fluctuation exceeds a specified spacetime parameter, the ultralogic ${}^*\mathbf{S}$ acts upon the w' and a universe is generated. All of the universes that are generated by this process are disjoint. 2 JAN 1999]

[Note (4): Relative to the deBroglie statement and to all the areas that appear to be humanly comprehensible, with the exception of an additional postulate relative to the continuous energy spectrum for electromagnetic radiation that is required for properton generation, the GGU-model is based upon one and only one postulate. The postulate is that how nature combines together natural laws and processes to bring into being a natural event or a change in a natural system is modeled or mirrored by the mental processes human beings use to describe the produced events or changes. This hypothesis is verified by the largest amount of empirical evidence that could ever exist since whenever a scientist discusses or predicts natural system behavior human mental processes are applied. This includes all of the methods used to gather and analyze evidence that would tend to verify any scientific hypothesis. 3 JAN 1999]

In the next section, the references, when they have been reproduced, appear at the end of each "chapter."

Important Note

In the mathematics section 3, objects are chosen that seem to yield the simplest possible entities. This is done to minimize controversy and to allow most conclusions to be established in a convincing and straightforward manner. The philosophy of science employed is the exact same philosophy of science used in theoretical cosmology and quantum logic investigations. In almost all cases, the physical-like interpretations correlate directly to the mathematical structure. The first section of part 3 is a reproduction of a Kobe Math. J. paper. The second section of part 3 is a reproduction of portions of the book *Ultralogics and More* (see reference [2] of part 1) where all the details for the actual construction of the nonstandard model can be found. It should be self-evident that the results contained within this monograph are only the must basic and that further in-depth investigations should be pursued.

Intuitively, I am confident that all of the theorems are correct. If any "proof" is not convincingly established, then this should be easy to rectify. Finally, I do not contend that this is THE solution to this problem and questions. Although these results are speculative in character, they are no more hypothetical than the Everett-Wheeler-Graham many-worlds interpretation or Hartle-Hawking quantum gravity model. I do contend that since the method was devised in 1979 that these results are the FIRST such solutions obtained scientifically. The term "scientifically" refers to the sciences of concrete mathematical modeling, mathematical logic and interpreting mathematical structures physically.

3. **The Mathematics**

The following is a direct copy, with numerously many printers errors corrected, of the indicated portions of the indicated published paper.

The following is a direct copy of this published paper, but with numerously many printer's errors corrected and with a few additional remarks (18 NOV 2010).

Herrmann, **R. A.**
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NONSTANDARD CONSEQUENCE OPERATORS

By Robert A. Herrmann

(Dedicated to Professor K. Iséki)

(Receiver October 22, 1984)

1. Introduction

In 1963, Abraham Robinson applied his newly discovered nonstandard analysis to formal first-order languages and developed a nonstandard logic [11] relative to the "truth" concept and structures. Since that time not a great deal of fundamental research has been attempted in this specific area with one notable exception [3]. However, when results from this discipline are utilized they have yielded some highly significant and important developments such as those obtained by Henson [4].

The major purpose for this present investigation is to institute formally a more general study than previously pursued. In particular, we study nonstandard logics relative to consequence operators [2] [6] [12] [13] defined on a nonstandard language. Since the languages considered are not obtained by the usual constructive methods, then this will necessitate the construction of an entirely new foundation distinctly different from Robinson's basic embedding techniques. Some very basic results of this research were very briefly announced in a previous report [6].

In order to remove ambiguity from the definition of the "finite" consequence operator, the definition of "finite" is the ordinary definition in that the empty set is finite and any nonempty set A is finite if and only if there exists a bijection $\mathrm{f}\colon \mathrm{A} \to [1, \mathrm{n}]$, where $[1, \mathrm{n}] = \{\mathrm{x} \mid \mathrm{n} \in \mathbb{N},\ 1 \leq \mathrm{x} \leq \mathrm{n}\}$ ($\mathbb{N}$ is the set of natural numbers with zero). Unless otherwise stated, all sets B that are infinite will also be assumed to be Dedekind-infinite. This occurs when a set B is denumerable, since B inherits a well-ordering from $\mathbb{N}$, or B is well-ordered [2, p. 248], or the Axiom of Choice is assumed. We note that within mathematics one is always allowed to make a finite choice from finitely many nonempty sets, among others [9, p. 1].

In 2, we give the basic definitions, notations and certain standard results are obtained that indicate the unusual behavior of the algebra of all consequence operators defined on a set. In 4, some standard properties relative to subalgebras and chains in the set of all consequence operators are investigated. Finally, the entire last section is devoted to the foundations of the theory of nonstandard consequence operators defined on a nonstandard language.

2. Basic concepts

Our notations and definitions for the standard theory of consequence operators are taken from references [2][6][12][13], and we now recall the most pertinent of these. Let L be any nonempty set that is often called a *language*, $\mathcal{P}(\mathrm{L})$ denote the power set of L and for any set X let $F(\mathrm{X})$ denote the finite power set of X (i.e. the set of all finite subsets of X.)

DEFINITION 2.1 A mapping $\mathrm{C}\colon \mathcal{P}(\mathrm{L}) \to \mathcal{P}(\mathrm{L})$ is a consequence operator (or closure operator) if for each $\mathrm{X},\ \mathrm{Y} \in \mathcal{P}(\mathrm{L})$

(i) $\mathrm{X} \subset \mathrm{C}(\mathrm{X}) = \mathrm{C}(\mathrm{C}(\mathrm{X})) \subset \mathrm{L}$ and if

(ii) $\mathrm{X} \subset \mathrm{Y}$, then $\mathrm{C}(\mathrm{X}) \subset \mathrm{C}(\mathrm{Y})$.

A consequence operator C defined on L is said to be *finite* (*finitary*, or *algebraic*) if it satisfies

(iii) $\mathrm{C}(\mathrm{X}) = \cup\{\mathrm{C}(\mathrm{A}) \mid \mathrm{A} \in \mathrm{F}(\mathrm{X})\}$.

REMARK 2.2 The above axioms (i) (ii) (iii) are not independent. Indeed, (i)(iii) imply (ii).

Throughout this entire article the symbol "C" with or without subscripts or with or without symbols juxtapositioned to the right will always denote a consequence operator. The only other symbols that will denote consequence operators are "I" and "U". The symbol $\mathcal{C}$ [resp. $\mathcal{C}_f$] denotes the set of all consequence operators [resp. finite consequence operators] defined on $\mathcal{P}(\mathrm{L})$.

DEFINITION 2.3. (i) Let I denote the identity map defined on $\mathcal{P}(\mathrm{L})$.
(ii) Let $\mathrm{U}\colon \mathcal{P}(\mathrm{L}) \to \mathcal{P}(\mathrm{L})$ be defined as follows: for each $\mathrm{X} \in \mathcal{P}(\mathrm{L}),\ \mathrm{U}(\mathrm{X}) = \mathrm{L}$.
(iii) For each $\mathrm{C}_1,\ \mathrm{C}_2 \in \mathcal{C}$, define $\mathrm{C}_1 \leq \mathrm{C}_2$ iff $\mathrm{C}_1(\mathrm{X}) \subset \mathrm{C}_2(\mathrm{X})$ for each $\mathrm{X} \in \mathcal{P}(\mathrm{L})$. (Note that $\leq$ is obviously a partial order defined on $\mathcal{C}$.)
(iv) For each $\mathrm{C}_1,\ \mathrm{C}_2 \in \mathcal{C}$, define $\mathrm{C}_1 \vee \mathrm{C}_2\colon \mathcal{P}(\mathrm{L}) \to \mathcal{P}(\mathrm{L})$ as follows: for each $\mathrm{X} \in \mathcal{P}(\mathrm{L}),\ (\mathrm{C}_1 \vee \mathrm{C}_2)(\mathrm{X}) = \mathrm{C}_2(\mathrm{X}) \cup \mathrm{C}_2(\mathrm{X})$.
(v) For each $\mathrm{C}_1,\ \mathrm{C}_2 \in \mathcal{C}$, define $\mathrm{C}_1 \wedge \mathrm{C}_2\colon \mathcal{P}(\mathrm{L}) \to \mathcal{P}(\mathrm{L})$ as follows: for each $\mathrm{X} \in \mathcal{P}(\mathrm{L}),\ (\mathrm{C}_2 \wedge \mathrm{C}_2)(\mathrm{X}) = \mathrm{C}_1(\mathrm{X}) \cap \mathrm{C}_2(\mathrm{X})$.
(vi) For each $\mathrm{C}_1,\ \mathrm{C}_2 \in \mathcal{C}$ define $\mathrm{C}_2 \vee_w \mathrm{C}_2\colon \mathcal{P}(\mathrm{L}) \to \mathcal{P}(\mathrm{L})$ as follows: for each $\mathrm{X} \in \mathcal{P}(\mathrm{L}),\ (\mathrm{C}_1 \vee_w \mathrm{C}_2)(\mathrm{X}) = \cap\{\mathrm{Y} \mid \mathrm{X} \subset \mathrm{Y} \subset \mathrm{L} \text{ and } \mathrm{Y} = \mathrm{C}_1(\mathrm{Y}) = \mathrm{C}_2(\mathrm{Y})\}$.

Prior to defining certain special consequence operators notice that $\mathrm{I},\ \mathrm{U} \in \mathcal{C}_f$ and that I [resp. U] is a lower [resp. upper] unit for the algebras $\langle \mathcal{C}, \leq \rangle$ and $\langle \mathcal{C}_f, \leq \rangle$.

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4. Nonstandard Consequence Operators

Let $\mathcal{A}$ be a nonempty finite set of symbols. It is often convenient to assume that $\mathcal{A}$ contains a symbol that represents a blank space. As usual any nonempty finite string of symbols from $\mathcal{A}$, with repetitions, is called a *word* [10, p.222]. A word is also said to be an (intuitive) *readable sentence* [5, p. 1]. We let W be the intuitive set of all words created from the *alphabet* $\mathcal{A}$. Note that in distinction to the usual approach, W does not contain a symbol for the empty word.

We accept the concept delineated by Markov [4], the so-called "abstraction of identity," and say that $\mathrm{w}_1,\ \mathrm{w}_2 \in \mathrm{W}$ are "equal" if they are composed of the same symbols written in the same intuitive order (left to right). The *join* or juxtaposition operation between $\mathrm{w}_1,\ \mathrm{w}_2 \in \mathrm{W}$ is the concept that yields the string $\mathrm{w}_1\mathrm{w}_2$ or $\mathrm{w}_2\mathrm{w}_1$. Thus W is closed under join. Notice that we may consider a denumerable formal language as a subset of W. (By adjoining a new symbol not in $\mathcal{A}$ and defining it as the unit, W becomes a free monoid generated by the set $\mathcal{A} \cup \{\text{new symbol}\}$.)

Since W is denumerable, then there exists an injection $i\colon \mathrm{W} \to \mathbb{N}$. Obviously, if we are working with a formal language that is a subset of W, then we may require i restricted to a formal language to be a Gödel numbering. Due to the join operation, a fixed member of W that contains two or more distinct symbols can be represented by various *subwords* that are joined together to yield the given fixed word. The word "mathematics" is generated by the join of $\mathrm{w}_1 = \mathrm{math},\ \mathrm{w}_2 = \mathrm{e},\ \mathrm{w}_4 = \mathrm{mat},\ \mathrm{w}_4 = \mathrm{ics}$. This word can also be formed by joining together 11 not necessarily distinct members of W.

Let $i[\mathrm{W}] = T$ and for each $n \in \mathbb{N}$, let $T^n = T^{[0,n]}$ denote the set of all mappings from $[0,n]$ into T. Each element of T^n is called a *partial sequence*, even though this definition is a slight restriction of the usual one that appears in the literature. Let $f \in T^n, n > 0$. Then the *order induced by* f is the simple inverse order determined by f applied to the simple order on $[0,n]$. Formally, for each $f(j),\ f(k) \in f[[0,n]]$, define $f(k) \leq_f f(j)$ iff $j \leq k$, where $\leq$ is the simple order for $\mathbb{N}$ restricted to $[0,n]$. In general, we will not use this notation $\leq_f$ but rather we will indicate this (finite) order in the usual acceptable manner by writing the symbols $f(n), f(n-1), \ldots, f(0)$ from left to right . Thus we symbolically let $f(n) \leq_f f(n-1) \leq_f \cdots \leq_f f(0) = f(n)f(n-1)\ldots f(0)$.

Let $f \in t^n$. Define $\mathrm{w}_f \in \mathrm{W}$ as follows: $\mathrm{w}_f = (i^{-1}(f(n)))(i^{-1}(f(n-1))) \cdots (i^{-1}(f(0)))$, where the operation indicated by juxtaposition is the join. We now define a relation on $P = \cup\{T^n \mid n \in \mathbb{N}\}$ as follows: let $f,\ g \in P$. Then for $f \in T^n$ and $g \in T^m$, define $f \sim g$ iff $(i^{-1}(f(n))) \cdots (i^{-1}f(0))) = (i^{-1}(g(m))) \cdots (i^{-1}(g(0)))$. It is obvious that $\sim$ is an equivalence relation on P. For each $f \in P$, $[f]$ denotes the equivalence class under $\sim$ that contains f. Finally, let $\mathcal{E} = \{[f] \mid f \in P\}$. Observe that for each $[f] \in \mathcal{E}$ there exist $f_0,\ f_m \in [f]$ such that $f_0 \in T^0,\ f_m \in T^m$ and if there exists some $k \in \mathbb{N}$ such that $0 < k < m$, then there exists some $g_k \in [f]$ such that $g_k \in T^k$ and if $j \in \mathbb{N}$ and $j > m$, then there does not exist $g_j \in T^j$ such that $g_j \in [f]$. If we define the *size* of a word $\mathrm{w} \in \mathrm{W}$ (size(w)) to be the number of not necessarily distinct symbols counting left to right that appear in W, then the size(w) $= m+1$. For each $\mathrm{w} \in \mathrm{W}$, there is $f_0 \in T^0$ such that $\mathrm{w} = i^{-1}(f_0(0))$ and such an $f_m \in [f_0]$ such that size (w) $= m+1$. On the other hand, given $f \in P$, then there is a $g_0 \in [f]$ such that $(i^{-1}(g_0(0))) \in \mathrm{W}$. Of course, each $g \in [f]$ is interpreted to be the word $(i^{-1}(g(k)))\ldots(i^{-1}(g(0)))$.

Each $[f] \in \mathcal{E}$ is said to be a (formal) word or (formal) *readable sentence.* All the intuitive concepts, definitions and results relative to consequence operators defined for A $\in \mathcal{P}$(W) are now passed to $\mathcal{P}(\mathcal{E})$ by means the map $\theta(i(\mathrm{w})) = [f_0]$. In the usual manner, the map θ is extended to subsets of each A $\in \mathcal{P}$(W), n-ary relations and the like. For example, let w $\in$ A $\in \mathcal{P}$(w). Then there exists $f_w \in P$ such that $f_w \in T^0$ and $f_w(0) = i(\mathrm{w})$. Then $\theta(i(\mathrm{w})) = [f_w]$. In order to simplify notation, the images of the extended $(\theta\, i)$ composition will often be indicated by bold notation with the exception of customary relation symbols which will be understood relative to the context. For example, if S is a subset of W, then we write $\theta(i[\mathrm{S}]) = \mathbf{S}$. [(12/27/12) Notice that $\mathbf{W} = \mathcal{E}$.

Let $\mathcal{N}$ be a superstructure constructed from the set W $\cup$ $\mathbb{N}$ as its set of atoms. (12/16/12 the ground set has now been so expanded so any symbols in W that are used for natural numbers are different from those in $\mathbb{N}$.) Our standard structure is $\mathcal{M} = (\mathcal{N}, \in, =)$. Let ${}^*\mathcal{M} = ({}^*\mathcal{N}, \in, =)$ be a nonstandard and elementary extension of $\mathcal{M}$. Further, ${}^*\mathcal{M}$ is an enlargement.

For an alphabet $\mathcal{A}$, there exists $[g] \in {}^*\mathcal{E} - \mathcal{E}$ such that there are only finitely many standard members of $\mathbb{N}$ in the range of g and these standard members injectively correspond to alphabet symbols in $\mathcal{A}$ [5, p. 24]. On the other hand, there exist $[g'] \in {}^*\mathcal{E} - \mathcal{E}$ such that the range of $[g']$ does not correspond in this manner to elements in $\mathcal{A}$ [5, p. 90].

Let $C \in \mathcal{H}$ map a family of sets $\mathcal{B}$ into $\mathcal{B}_0$. If C satisfies either the Tarski axioms (i), (ii) or (i), (iii), or the *-transfer *(i), *(ii), or *(i), *(iii) of these axioms, then C is called a *subtle consequence operator*. For example, if C $\in \mathcal{C}$, then it is immediate that ${}^*\mathbf{C}\colon {}^*(\mathcal{P}(\theta(\mathrm{A})) \to {}^*\mathcal{P}(\theta(\mathrm{A})))$ satisfies *(i) and *(ii) for the family of all internal subsets of ${}^*(\theta(\mathrm{A}))$. This ${}^*\mathbf{C}$ is a subtle consequence operator. For any set $A \in \mathcal{N}$, let ${}^\sigma A = \{\, {}^*a \mid a \in A\}$. (In general, this definition does not correspond to that used by other authors.) If for a subtle consequence operator C there does not exist some similarly defined $D \in \mathcal{N}$ such that $C = {}^\sigma\mathbf{D}$ or $C = {}^*\mathbf{D}$, then C is called a *purely* subtle consequence operator. Let infinite $A \subset \mathcal{E}$ and $B = {}^*A - {}^\sigma A$. Then the identity $I\colon \mathcal{P}(B) - \mathcal{P}(B)$ is a purely subtle consequence operator.

There are certain technical procedures associated with the σ map that take on a specific significance for consequence operators. Recall that $\mathcal{N}$ is closed under finitely many power set or finite power set iterations. Let $X,\ Y \in \mathcal{N}$. It is not difficult to show that if $\mathcal{P}\colon \mathcal{P}(X) \to Y$, then for each $A \in \mathcal{P}(X)$, ${}^*(\mathcal{P}(A)) = {}^*\mathcal{P}({}^*A)$. Moreover, if $F\colon \mathcal{P}(X) \to Y$, where F is the finite power set operator, then for each $A \in \mathcal{P}(A)$, ${}^*(F(A)) = {}^*F({}^*A)$. If C $\in \mathcal{C}$ and $X \subset$ W, then $\mathbf{C}\colon \mathcal{P}(X) \to \mathcal{P}(X)$ has the property that for each $A \in \mathcal{P}(X)$, ${}^*(\mathbf{C}(A)) = {}^*\mathbf{C}({}^*A)$.

Recall that we identify each ${}^*n \in {}^*\mathbb{N}$ with $n \in \mathbb{N}$ since ${}^*n$ is but a constant sequence with the value n. Utilizing this fact, we have the following straightforward lemma the proof of which is omitted.

LEMMA 4.1.

(i) *Let* $A \in \mathcal{N}$. *Then* ${}^\sigma(F(A)) = F({}^\sigma A)$. *If also* $A \subset (\mathcal{W} \cup \mathcal{E})$, *then* ${}^\sigma(F(A)) = F(A)$.

(ii) *Let* C $\in \mathcal{C}'$, B $\subset$ X $\subset \mathcal{W}$.

(a) ${}^\sigma(\mathbf{C}(\mathbf{B})) = \mathbf{C}(\mathbf{B})$.

(b) ${}^*\mathbf{C} \bigm| \{ {}^*\mathbf{A} \mid \mathbf{A} \in \mathcal{P}(X)\} = \{({}^*\mathbf{A},\ {}^*\mathbf{B}) \mid (\mathbf{A},\mathbf{B}) \in \mathbf{C}\} = {}^\sigma\mathbf{C}$.

(c) *If* $\mathrm{F} \in \mathrm{F(B)}$, *then* ${}^\sigma(\mathbf{C}(\mathbf{F})) \subset ({}^\sigma\mathbf{C})({}^\sigma\mathbf{F}) = ({}^\sigma\mathbf{C})(\mathbf{F})$. *Also* ${}^\sigma(\mathbf{C}(\mathbf{B})) \subset ({}^\sigma\mathbf{C})({}^*\mathbf{B})$ *and, in general,* ${}^\sigma(\mathbf{C}(\mathbf{B})) \neq ({}^\sigma\mathbf{C})({}^*\mathbf{B})$, ${}^\sigma(\mathbf{C}(\mathbf{F})) \neq ({}^\sigma\mathbf{C})(F)$.

(d) *If* $C \in \mathcal{C}'_f$, *then* ${}^\sigma(\mathbf{C}(\mathbf{B})) = \bigcup\{{}^\sigma(\mathbf{C}(\mathbf{F})) \mid \mathbf{F} \in {}^\sigma(F(\mathbf{B}))\} = \bigcup\{{}^\sigma(\mathbf{C}(\mathbf{F})) \mid \mathbf{F} \in F({}^\sigma\mathbf{B})\} = \bigcup\{\mathbf{C}(\mathbf{F}) \mid \mathbf{F} \in F(\mathbf{B})\}$.

(A duplicate lemma holds, where $A \in \mathcal{N}$ and $C \in \mathcal{C}$, where $\mathcal{C}$ set of consequence operators defined on subsets of W if W is included as a subset of the ground set. The difference is that the "bold" notion does not appear.)

Throughout the remainder of this paper, we remove from $\mathcal{C}$ the one and only one inconsistent consequence operator U. Thus notationally we let $\mathcal{C}$ denote the set of all consequence operators defined on infinite $\mathrm{L} = \mathrm{X} \subset \mathrm{W}$ with the exception of U. Two types of chains will be investigated. Let T be any chain in $\langle\mathcal{C},\leq\rangle$ and T' be any chain with the additional property that for each $\mathrm{C} \in \mathrm{T}'$ there exists some $\mathrm{C}' \in \mathrm{T}'$ such that $\mathrm{C} < \mathrm{C}'$.

THEOREM 4.2 *There exists some* $C_0 \in {}^*\mathbf{T}$ *such that for each* $\mathrm{C} \in \mathrm{T}$, ${}^*\mathbf{C} \leq C_0$. *There exists some* $C'_0 \in {}^*\mathbf{T}'$ *such that* C'_0 *is a purely subtle consequence operator and for each* $\mathrm{C} \in \mathrm{T}'$, ${}^*\mathbf{C} < C'_0$. *Each member of* ${}^*\mathbf{T}$ *and* ${}^*\mathbf{T}'$ *are subtle consequence operators.*

PROOF. Let $R = \{(x,y) \mid x,\ y \in \mathbf{T}\}$ and $x \leq y\}$ and $R'\{(x,y) \mid x,\ y \in \mathbf{T}'$ and $x < y\}$. In the usual manner, it follows that R and R' are concurrent on the set $\mathbf{T}$ and $\mathbf{T}'$ respectively. Thus there is some $C_0 \in {}^*\mathbf{T}$ and $C'_0 \in {}^*\mathbf{T}'$ such that for each $\mathrm{C} \in \mathrm{T}$ and $\mathrm{C}' \in \mathrm{T}$, ${}^*\mathbf{C} \leq C_0$ and ${}^*\mathbf{C}' < C'_0$ since ${}^*\mathcal{M}$ is an enlargement. Note that the members of ${}^*\mathbf{T}$ and ${}^*\mathbf{T}'$ are defined on the set of all internal subsets of ${}^*\mathbf{L}$. However, if there is some similarly defined $D \in \mathcal{N}$ such that C_0 or $C'_0 = {}^\sigma D$, then since ${}^\sigma D$ is only defined for *-extensions of the (standard) members of $\mathcal{P}(\mathrm{L})$ and each $E \in {}^*\mathbf{T}$ or ${}^*\mathbf{T}'$ is defined on the internal subsets of ${}^*\mathbf{L}$ and there are internal subsets of ${}^*\mathbf{L}$ that are not *-extensions of standard sets we would have a contradiction. Of course, each member of ${}^*\mathbf{T}$ or ${}^*\mathbf{T}'$ is a subtle consequence operator. Hence each $E \in {}^*\mathbf{T}$ or ${}^*\mathbf{T}'$ is either equal to some ${}^*\mathbf{C}$, where $\mathrm{C} \in \mathrm{T}$ or $\mathrm{C} \in \mathrm{T}'$ or it is a purely subtle consequence operator. Now there does not exist a $D \in \mathcal{N}$ such that $C'_0 = {}^*\mathbf{D}$ since $C'_0 \in {}^*\mathbf{T}'$ and ${}^*\mathbf{C} \neq C'_0$ for each ${}^*\mathbf{C} \in {}^\sigma\mathbf{T}'$ would yield the contradiction that ${}^*D \in {}^*\mathbf{T}' - {}^\sigma\mathbf{T}'$ but ${}^*\mathbf{D} \in {}^\sigma\mathcal{C}$. Hence C'_0 is a purely subtle consequence operator. This completes the proof.

Let $\mathrm{C} \in \mathrm{T}'$. Since ${}^*\mathbf{C} < C'_0$, then C'_0 is "more powerful" than any $\mathrm{C} \in \mathrm{T}'$ in the following sense. If $\mathrm{B} \in \mathcal{P}(\mathrm{L})$, then for each $\mathrm{C} \in \mathrm{T}'$ it follows that $\mathbf{C}(\mathbf{B}) \subset {}^*(\mathbf{C}(\mathbf{B})) = {}^*\mathbf{C}({}^*\mathbf{B}) \subset C'_0({}^*\mathbf{B})$. Recall that, for $\mathrm{C} \in \mathcal{C}$, a set $\mathrm{B} \in \mathcal{P}(\mathrm{L})$ is called a *C-deductive system* if $\mathrm{C(B)} = \mathrm{B}$. From this point on, all results are restricted to chains in $\langle\mathcal{C}_f,\leq\rangle$.

THEOREM 4.3. *Let* $\mathrm{C} \in \mathrm{T} \cup \mathrm{T}'$ *and* $\mathrm{B} \in \mathcal{P}(\mathrm{L})$. *Then there exists a *-finite* $F_0 \in {}^*(F(\mathbf{B}))$ *such that* $\mathbf{C}(\mathbf{B}) \subset {}^*\mathbf{C}(F_0) \subset {}^*\mathbf{C}({}^*\mathbf{B}) = {}^*(\mathbf{C}(\mathbf{B}))$ *and* ${}^*\mathbf{C}(F_0) \cap \mathbf{L} = \mathbf{C}(\mathbf{B}) = {}^*\mathbf{C}(F_0) \cap \mathbf{C}(\mathbf{B})$.

PROOF. Consider the binary relation $Q = \{(x,y) \mid x \in \mathbf{C}(\mathbf{B}),\ y \in F(\mathbf{B})$ *and* $x \in \mathbf{C}(y)\}$. By axiom (iii), the domain of Q is $\mathbf{C}(\mathbf{B})$. Let

$(x_1,y_1),\ldots,(x_n,y_n)\in Q$. By theorem 1 in [6, p. 64], (the monotone theorem) we have that $\mathbf{C}(y_1)\cup\cdots\cup\mathbf{C}(y_n)\subset\mathbf{C}(y_1\cup\cdots\cup y_n)$. Since $F=y_1\cup\cdots\cup y_n\in F(\mathbf{B})$, then $(x_1,F),\ldots,(x_n,F)\in Q$. Thus Q is concurrent on $\mathbf{C}(\mathbf{B})$. Hence there exists some $F_0\in {}^*(F(\mathbf{B}))$ such that ${}^\sigma(\mathbf{C}(\mathbf{B}))=\mathbf{C}(\mathbf{B})\subset {}^*\mathbf{C}(F_0)\subset {}^*\mathbf{C}({}^*\mathbf{B})= {}^*(\mathbf{C}(\mathbf{B}))$. Since ${}^\sigma\mathbf{L}=\mathbf{L}$, then ${}^*\mathbf{C}(F_0)\cap\mathbf{L}=\mathbf{C}(\mathbf{B})={}^*\mathbf{C}(F_0)\cap\mathbf{C}(\mathbf{B})$.

COROLLARY 4.3.1 *If* $\mathrm{C}\in\mathcal{C}_f$ *and* $\mathrm{B}\in\mathcal{P}(\mathrm{L})$ *is a C-deductive system, then there exists a *-finite* $F_0\subset {}^*\mathbf{B}$ *such that* ${}^*\mathbf{C}(F_0)\cap\mathbf{L}=\mathbf{B}$.

PROOF. Simply consider the one element chain $\mathrm{T}=\{\mathrm{C}\}$.

COROLLARY 4.3.2. *Let* $\mathrm{C}\in\mathcal{C}_f$. *There there exists a *-finite* $F_1\subset {}^*\mathbf{L}$ *such that for each C-deductive system* $\mathrm{B}\subset\mathrm{L}$, ${}^*\mathbf{C}(F_1)\cap\mathbf{B}=\mathbf{B}$.

PROOF. Let $\mathrm{T}=\{\mathrm{C}\}$ and the "B" in theorem 4.3 equal L. The result now follows in a straightforward manner.

THEOREM 4.4. *Let* $\mathrm{B}\in\mathcal{P}(\mathrm{L})$.

(i) *There exists a *-finite* $F_B\in {}^*(F(\mathbf{B}))$ *and a subtle consequence operator* $C_B\in {}^*\mathbf{T}$ *such that for all* $\mathrm{C}\in\mathrm{T}$, ${}^\sigma(\mathbf{C}(\mathbf{B}))=\mathbf{C}(\mathbf{B})\subset C_B(F_B)$.

(ii) *There exists a *-finite* $F_B\in {}^*(F(\mathbf{B}))$ *and a purely subtle consequence operator* $C'_B\in {}^*\mathbf{T}'$ *such that for all* $\mathrm{C}\in\mathrm{T}'$, ${}^\sigma(\mathbf{C}(\mathbf{B}))=\mathbf{C}(\mathbf{B})\subset C'_B(F_B)$.

PROOF. Consider the "binary" relation $Q=\{((x,z),(y,w))\mid x\in\mathbf{T},\ y\in\mathbf{T},\ w\in F(\mathbf{B}),\ z\in x(w),\ z\in\mathcal{P}(\mathbf{L}), z\in x(w),\ \text{and}\ x(w)\subset y(w)\}$. Let $\{((x_1,z_1),(y_1,w_1)),\ldots,((x_n,z_n),(y_n,w_n))\}\subset Q$. Notice that $F=w_1\cup\cdots\cup w_n\in F(\mathbf{B})$ and for the set $K=\{x_1,\ldots,x_n\}$, let D be the largest member of K with respect to $\leq$. It follows that $z_i\in x_i(w_i)\subset x_i(F)\subset D(F)$ for each $i=1,\ldots,n$. Hence $\{((x_1,z_1),(D,F)),\ldots,((x_n,z_n),(D,F))\}\subset Q$ implies that Q is concurrent on its domain. Consequently, there exists some $(C_B,F_B)\in {}^*\mathbf{T}\times {}^*(F(\mathbf{B}))$ such that for each $(x,z)\in$ domain of Q, $({}^*(x,z),(C_B,F_B))\in {}^*Q$. Or, for each $(u,v)\in {}^\sigma$(domain of Q), $((u,v),(C_B,F_B))\in {}^*Q$. Let arbitrary $\mathrm{C}\in\mathrm{T}$ and $b\in C(B)$. Then there exists some $F'\in F(\mathbf{B})$ such that $\mathbf{b}\in\mathbf{C}(F')$. Thus $({}^*\mathbf{C},\ {}^*\mathbf{b})\in {}^\sigma$(domain of Q). Consequently, for each $\mathrm{C}\in\mathrm{T}$ and $\mathbf{b}\in\mathbf{C}(\mathbf{B})$, $\mathbf{b}={}^*\mathbf{b}\in({}^*\mathbf{C})(F_B)\subset C_B(F_B)$. This all implies that for each $\mathrm{C}\in\mathrm{T}$, ${}^\sigma(\mathbf{C}(\mathbf{B}))=\mathbf{C}(\mathbf{B})\subset C_B(F_B)$.

(ii) Change the relation Q to Q' by requiring that $x\neq y$. Replace D in the proof of (i) above with D' is greater than and not equal to the largest member of K. Such a D' exists in $\mathbf{T}'$ from the definition of $\mathbf{T}'$. Continue the proof in the same manner in order to obtain C'_B and F'_B. The fact that C'_B is a purely subtle consequence operator follows in the same manner as in the proof of theorem 4.2.

COROLLARY 4.4.1 *There exists a* [*resp. purely*] *subtle consequence operator* $C_L\in {}^*\mathbf{T}$ [*resp.* ${}^*\mathbf{T}'$] *and a *-finite* $F_L\in {}^*(F(\mathbf{L}))$ *such that for all* $\mathrm{C}\in\mathrm{T}$ [*resp.* T'] *and each* $\mathrm{B}\in\mathcal{P}(\mathrm{L})$, $\mathbf{B}\subset\mathbf{C}(\mathbf{B})\subset C_L(F_L)$.

PROOF. Simply let "B" in theorem 4.4 be equal to L. Then there exists a [resp. purely] subtle $C_L\in {}^*\mathbf{T}$ [resp. ${}^*\mathbf{T}'$] and $F_L\in {}^*(F(\mathbf{L}))$ such that for all $\mathrm{C}\in\mathrm{T}$ [resp. T'] $\mathbf{C}(\mathbf{L})\subset C_L(F_L)$. If $\mathrm{B}\in\mathcal{P}(\mathrm{L})$ and $\mathrm{C}\in\mathrm{T}$ [resp. T'], then $\mathrm{B}\subset\mathrm{C}(\mathrm{B})\subset\mathrm{C}(\mathrm{L})$.

Thus for each $\mathrm{B} \in \mathcal{P}(\mathrm{L})$ and $\mathrm{C} \in \mathrm{T}$ [resp. T'] $\mathbf{B} \subset \mathbf{C}(\mathbf{B}) \subset C_L(F_L)$ and the theorem is established.

The nonstandard results in this section have important applications to mathematical philosophy. We present two such applications. Let $\mathcal{F}$ be the symbolic alphabet for any formal language L with the usual assortment of primitive symbols [10, p. 59]. We note that it is possible to mimic the construction of L within $\mathcal{E}$ itself. If this is done, then it is not necessary to consider the map and we may restrict our attention entirely to the sets $\mathcal{E}$ and ${}^*\mathcal{E}$.

Let S denote the predicate consequence operator by the standard rules for predicate (proof-theory) deduction as they appear on pages 59 and 60 of reference [10]. Hence $\mathrm{A} \in \mathcal{P}(\mathrm{L})$, $\mathrm{S(A)} = \{x \mid x \in \mathrm{L} \text{ and } \mathrm{A} \vdash x\}$. It is not difficult to restrict the modus ponens rule of inference in such a manner that a denumerable set $\mathrm{T}' = \{\mathrm{C}_n \mid n \in \mathbb{N}\}$ of consequence operators defined on $\mathcal{P}(\mathrm{L})$ is generated with the following properties.

(i) For each $\mathrm{A} \in \mathcal{P}(\mathrm{L})$, $\mathrm{S(A)} = \bigcup\{\mathrm{C}_n(\mathrm{A}) \mid n \in \mathbb{N}\}$ and $\mathrm{C}_n \neq \mathrm{S}$ for any $n \in \mathbb{N}$.

(ii) For each $\mathrm{C} \in \mathrm{T}'$ there is a C' such that $\mathrm{C} < \mathrm{C}'$ [5, p.57]. Let $\mathrm{A} \in \mathcal{P}(\mathrm{L})$ be any S-deductive system. The $\mathrm{A} = \mathrm{S(A)} = \bigcup\{\mathrm{C}_n(\mathrm{A}) \mid n \in \mathbb{N}\}$ yields that A is a C_n-deductive system for each $n \in \mathbb{N}$. Thus S and C_n $n \in \mathbb{N}$ are consequence operators defined on $\mathcal{P}(\mathrm{A})$ as well as on $\mathcal{P}(\mathrm{L})$.

THEOREM 4.5. *Let* L *be a first-order language and* $\mathrm{A} \in \mathcal{P}(\mathrm{L})$. *Then there exists a purely subtle* $C_1 \in {}^*\mathrm{T}'$ *and a *-finite* $F_1 \in {}^*(F(\mathbf{A}))$ *such that for each* $\mathrm{B} \in \mathcal{P}(\mathrm{A})$ *and each* $\mathrm{C} \in \mathrm{T}'$

(i) $\mathbf{C}(\mathbf{B}) \subset C_1(F_1)$,

(ii) $\mathbf{S}(\mathbf{B}) \subset C_1(F_1) \subset {}^*\mathbf{S}(F_1) \subset {}^*(\mathbf{S}(\mathbf{A}))$.

(iii) ${}^*\mathbf{S}(F_1) \cap \mathbf{L} = \mathbf{S}(\mathbf{A}) = C_1(F_1) \cap \mathbf{L}$.

PROOF. The same proof as for corollary 4.4.1 yields that there is some purely subtle $C_1 \in {}^*\mathbf{T}'$ and $F_1 \in {}^*(F(\mathbf{A}))$ such that for each $\mathrm{B} \in \mathcal{P}(\mathrm{A})$ and each $\mathrm{C} \in \mathrm{T}'$, $\mathbf{C}(\mathbf{B}) \subset C_1(F_1)$ and (i) follows. From (i), it follows that $\bigcup\{{}^\sigma(\mathbf{C}(\mathbf{B})) \mid \mathrm{C} \in \mathrm{T}'\} = \bigcup\{\mathbf{C}(\mathbf{B}) \mid \mathrm{C} \in \mathrm{T}'\} = \mathbf{S}(\mathbf{B}) = {}^\sigma(\mathbf{S}(\mathbf{B})) \subset C_1(F_1)$ and the first part of (ii) holds. By *-transfer $C_1 < {}^*\mathbf{S}$ and C_1 and ${}^*\mathbf{S}$ are defined on internal subsets of ${}^*\mathbf{A}$. Thus $C_1(F_1) \subset {}^*\mathbf{S}(F_1) \subset {}^*\mathbf{S}({}^*\mathbf{A}) = {}^*(\mathbf{S}(\mathbf{A}))$ by the *-monotone property. This completes (ii). Since $\mathbf{S}(\mathbf{A}) \subset C_1(F_1) \subset {}^*\mathbf{S}(F_1) \subset {}^*(\mathbf{S}(\mathbf{A}))$ from (ii), then (iii) follows and the theorem is proved.

REMARK 4.6. Of course, it is well known that there exists some $F \in {}^*(F(\mathbf{A}))$ such that $\mathbf{S}(\mathbf{A}) \supset \mathbf{A} \subset F \subset {}^*\mathbf{A}$ and *-transfer of axiom (i) yields that ${}^*\mathbf{S}(F) \subset {}^*\mathbf{S}({}^*\mathbf{A}) = {}^*(\mathbf{S}(\mathbf{A}))$. However, F_1 of theorem 4.5 is of a special nature in that the purely subtle C_1 applied to F_1 yields the indicated properties. Also theorem 4.5 holds for many other infinite languages and deductive processes.

Let L be a language and let M be a structure in which L can be interpreted in the usual manner. A consequence operator C is *sound* for M if whenever $\mathrm{A} \in \mathcal{P}(\mathrm{L})$ has the property that $M \models \mathrm{A}$, then $M \models \mathrm{C(A)}$. As usual, $T(M) = \{x \mid$

$x \in$ L and $M \models x\}$. Obviously, if C is sound for M, then $T(M)$ is a C-deductive system.

Corollary 4.3.1 implies that there exists *-finite $F_0 \subset {}^*(T(M))$ such that ${}^*\mathbf{C}(F_0) \cap \mathbf{L} = T(M)$. Notice that the fact that F_0 is *-finite implies that F_0 is *-recursive. Moreover, trivially, F_0 is a *-axiom system for ${}^*\mathbf{C}(F_0)$, and we do not lack knowledge about the behavior of F_0 since any formal property about $\mathbf{C}$ or recursive sets, among others, must hold for ${}^*\mathbf{C}$ or F_0 when property interpreted. If L is a first-order language, then S is sound for first-order structures. Theorem 4.5 not only yields a *-finite F_1 but a purely subtle consequence operator C_1 such that, trivially, F_1 is a *-axiom for $C_1(F_1)$ and for ${}^*\mathbf{S}(F_1)$. In this case, we have that ${}^*\mathbf{S}(F_1) \cap \mathbf{L} = T(M) = C_1(F_1) \cap \mathbf{L}$. By the use of internal and external objects, the nonstandard logics $\{{}^*\mathbf{C}, {}^*\mathbf{L}\}$, $\{C_1, {}^*\mathbf{L}\}$ and $\{{}^*\mathbf{S}, {}^*\mathbf{L}\}$ technically by-pass a portion of Gödel's first incompleteness theorem.

By definition b $\in$ S(B), B $\in \mathcal{P}$(L) iff there is a finite length "proof" of b from the premises B. It follows, that for each $b \in {}^*(T(M))$ there exists a *-finite length proof of b from a *-finite set of premises F_1.

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Robert A. Herrmann
Mathematics Department,
U. S. Naval Academy,
ANNAPOLIS, MD 21402
U. S. A.

The material that begins on the next page (23) is taken from "The Theory of Ultralogics" (Ultralogics and More) with a minor addition or two. Although much here continues to be applicable to the model, a new approach using the notion of instruction-information and logic-systems has been introduced in the appendix. In what follows, symbols have been changed. W $= \mathcal{W}$ and T $= \mathcal{E}$.

7.1 Introduction.

Consider the real line. If you believe that time is the ordinary continuum, then the entire real line can be your time line. Otherwise, you may consider only a subset of the real line as a time line. In the original version of this section, the time concept for the GGU-model was presented in a unnecessarily complex form. As shown in [3], one can assume an absolute substratum time within the NSP-world. It is the infinitesimal light-clock time measures that may be altered by physical processes. In my view, the theory of quantum electrodynamics would not exist without such a NSP-world time concept.

Consider a small interval $[a, b),\ a < b$ as our basic time interval where as the real numbers increase the time is intuitively considered to be increasing. In the following approach, one may apply the concept of the persistence of mental version relative to descriptions for the behavior of a Natural-system at a moment of time within this interval. An exceptionally small subinterval can be chosen within $[a, b)$ as a maximum subinterval length $= M$. "Time" and the size of a "time" interval as they are used in this and the following sections refer to an intuitive concept used to aid in comprehending the notation of an event sequence - an event ordering concept. First, let $a = t_0$. Then choose t_1 such that $a < t_1 < b$. There is a partition $t_1, \ldots, t_m$ of $[a, b)$ such that $t_0 < t_1 < \cdots < t_m < b$ and $t_{j+1} - t_j \leq M$. The final subinterval $[t_m, b)$ is now separated, by induction, say be taking midpoints, into an increasing sequence of times $\{t_q\}$ such that $t_m < t_q < b$ for each q and $\lim_{q\to\infty}\ t_q = b$.

Assume the prototype $[a, b)$ with the time subintervals as defined above. Let $[t_j, t_{j+1})$ be any of the time subintervals in $[a, b)$. For each such subinterval, let W_i denote the readable sentence

This|||frozen|||segment|||gives|||a|||description|||for|||the|||

time|||interval|||that|||has|||as|||its|||leftmost|||endpoint|||the

|||time|||$\lceil t_i \rceil$|||that|||corresponds|||to|||the|||natural|||number|||i.

Let $\mathrm{T}_i = \{\mathrm{xW}_i \mid \mathrm{x} \in \mathcal{W}\}$. The set T_i is called a *totality* and each member of any such T_i is called a *frozen segment.* Notice that since the empty word is not a member of $\mathcal{W}$, then the cardinality of each member of T_i is greater than that of W_i. Each T_i is a (Dedekind) denumerable set, and if $i \neq j$, then $\mathrm{T}_i \cap \mathrm{T}_j = \emptyset$.

W_i is only an identifier and may be altered. "Time," either its measure or otherwise, is not the actual underlying interpretation for these intervals. Time refers to an external event ordering concept, an intuitive *event sequence.* For most purposes, simply call these intervals "event intervals." [These event intervals can correspond to the universal event numbers concept.] In the above descriptions for W_i, simply replace "time|||interval" with "event|||interval" and replace the second instance of the word "time" with the word "event." If this event sequence interpretation is made, then other compatible interpretations would be necessary when applying some of the following results. ((Added 1/27/2015) The refined developmental paradigm approach in Herrmann (2013b) should be consulted. The notion of a "continuous" development is a matter of philosophic choice. As here presented in order to have a meaningful description, this approach can be considered as an exceptionally refined approximation for a continuous development. Or, a continuous development can be considered as an approximation for this development paradigm approach.)

I point out two minor aspects of the above constructions. First, within certain descriptions there are often "symbols" used for real, complex, natural numbers etc. These objects also exist as abstract objects within the structure $\mathcal{M}$. No inconsistent interpretations should occur when these objects are specifically modeled within $\mathcal{M}$ since to my knowledge all of the usual mathematical objects used within physical analysis are disjoint from $\mathcal{E}$ as well as disjoint from any finite Cartesian product of $\mathcal{E}$ with itself. If for future research within physical applications finite partial sequences of natural numbers and the finite equivalence classes that appear in $\mathcal{E}$ are needed and are combined into one model for different purposes than the study of descriptions, then certain modifications would need to be made so that interpretations would remain consistent. Secondly, I have tried whenever intuitive strings are used or sets of such strings are defined to use Roman letter notation for such objects. This only applies for the intuitive model. Also W_i is only an identifier and may be altered.

7.2 Developmental Paradigms

It is clear that if one considers a time interval of the type $(-\infty, +\infty)$, $(-\infty, b)$ or $[a, +\infty)$, then each of these may be considered as the union of a denumerable collection of time intervals of the type $[a, b)$ with common endpoint names displayed. Further, although $[a, b)$ is to be considered as subdivided into denumerably many subintervals, it is not necessary that each of the time intervals $[t_j, t_{j+1}) \subset [a, b)$ be accorded a corresponding description for the appearance of a specific Natural-system that is distinct from all others that occur throughout the time subinterval. Repeated descriptions only containing a different last natural number i in the next to last position will suffice. Each basic developmental paradigm will be restricted, at present, to such a time interval $[a, b)$.

Where human perception and descriptive ability is concerned, the least controversial approach would be to consider only finitely many descriptive choices as appropriate. A finite set is recursive and such a choice, since the result is such a set, would be considered to be the simplest type of algorithm. You "simply" check to see if an expression is a member of such a finite set. If we limited ourselves to finitely many human choices for Natural system descriptions from the set of all totalities and did not allow a denumerable or a continuum set to be chosen, then the next result establishes that within the Nonstandard Physical world (i.e. NSP-world) such a finite-type of choice can be applied and a continuum of descriptions obtained.

The following theorem is not insignificant even if we are willing to accept a denumerable set of distinct descriptions — descriptions that are not only distinct in the next to the last symbol, but are also distinctly different in other aspects as well. For, if this is the case, the results of Theorem 7.2.1 still apply. The same finite-type of process in the NSP-world yields such a denumerable set as well.

The term "NSP-world" will signify a certain second type of interpretation for nonstandard entities. In particular, the subtle logics, unreadable sentences, etc. This interpretation will be developed throughout the remainder of this book. One important aspect of how descriptions are to be interpreted is that a description correlates directly to an assumed or observed real Natural phenomenon, and conversely. In these investigations, the phenomenon is called an *event.*

In order to simplify matters a bit, the following notation is employed. Let $\mathcal{T} = \{\mathrm{T}_i \mid i \in \mathbb{N}\}$. Let $F(\mathcal{T})$ be the set of all **nonempty** and *finite* subsets of $\mathcal{T}$. This symbol has been used previously to include the empty set, this set is now excluded. Now let $\mathrm{A} \in F(\mathcal{T})$. Then there exists a finite choice set s such that $\mathrm{x} \in \mathrm{s}$ iff there exists a unique $\mathrm{T}_i \in \mathrm{A}$ and $\mathrm{x} \in \mathrm{T}_i$. Now let the set $\mathcal{C}$ denote the set of all such finite choice sets. As to interpreting these results within the NSP-world, the following is essential. Within nonstandard analysis the term "hyper" is often used for the result of the * map. For example, you have ${}^*\mathbb{R}$ as the hyperreals since $\mathbb{R}$ is termed the real numbers. For certain, but not all concepts, the term "hyper" or the corresponding * notation will be universally replaced by the term "ultra." Thus, certain purely subtle words or *-words become "ultrawords" within the developmental paradigm interpretation. [Note: such a word was previously called a superword.] Of course, for other scientific or philosophical systems, such abstract mathematical objects can be reinterpreted by an appropriate technical term taken from those disciplines.

As usual, we are working within any enlargement and all of the above intuitive objects are embedded into the G-structure. Recall, that to simplify expressions, we often suppress within our first-order statements a specific superstructure element that bounds a specific quantifier. [Note 2 MAY 1998: The material between the [[and the]] has been altered from the original that appeared in the 1993 revision.] [[Although theorem 7.2.1 may not be insignificant, it is also not necessary for the other portions of this research. The general axiom of choice can be applied to generate formally developmental paradigms.

The $\mathcal{S}$ in Theorem 7.2.1 determines a special intrinsic ultranatural selection process (i.e. **IUN-selection process**). This process is obtained by application of hyperfinite choice.

Theorem 7.2.1 *Let* $\emptyset \neq \gamma \subset \mathbb{N}$ *and* $\widetilde{\mathcal{T}} = \{\mathbf{T}_i \mid i \in \gamma\}$. *There exists a set of sets* $\mathcal{S}$ *determined by hyperfinite set* Q *and hyper finite choice defined on* Q *such that:*

(i) $s' \in \mathcal{S}$ *iff for each* $\mathbf{T} \in \widetilde{\mathcal{T}}$ *there is one and only one* $[g] \in {}^*\mathbf{T}$ *such that* $[g] \in s'$, *and if* $x \in s'$, *then there is some* $\mathbf{T} \in \widetilde{\mathcal{T}}$ *and some* $[g] \in {}^*\mathbf{T}$ *such that* $x = [g]$. *(If* ${}^*[g] \in {}^\sigma\mathbf{T}$, *then* $[g] = [f] \in \mathbf{T}$.)

Proof. (i) Let $A \in F(\widetilde{\mathcal{T}})$. Then from the definition of $\widetilde{\mathcal{T}}$, there exists some $n \in \mathbb{N}$ such that $A = \{\mathbf{T}_{j_i} \mid i = 0, \ldots, n \wedge j_i \in \mathbb{N}\}$. From the definition of $\mathbf{T}_k$, each $\mathbf{T}_k$ is denumerable. Notice that any $[f] \in \mathbf{T}_k$ is associated with a unique member of $i[\mathcal{W}]$. Simply consider the unique $f_0 \in [f]$. The unique member of $i[\mathcal{W}]$ is by definition $f_0(0)$. Thus each member of $\mathbf{T}_k$ can be specifically identified. Hence, for each $\mathbf{T}_i$ there is a denumerable $M_i \subset \mathbb{N}$ and a bijection $h_i\colon M_i \to \mathbf{T}_i$ such that $a_i \in \mathbf{T}_i$ iff there is a $k_i \in M_i$ and $h_i(k_i) = a_i$. Consequently, for each $i = 0, \ldots, n$ and $a_{j_i} \in \mathbf{T}_{j_i}$, we have that $h_{j_i}(k_{j_i}) = a_{j_i}$, and conversely for each $i = 0, \ldots, n$ and $k_{j_i} \in M_{j_i},\ h_{j_i}(k_{j_i}) \in \mathbf{T}_{j_i}$. Obviously, $\{h_{j_i}(k_{j_i}) \mid i = 0, \ldots, n\}$ is a finite choice set. All of the above may be translated into the following sentence that holds in $\mathcal{M}$. (Note: Choice sets are usually considered as the range of choice functions. Further, "bounded formula simplification" has been used.)

$$\begin{aligned} &\forall y(y \in F(\widetilde{\mathcal{T}}) \to \exists s((s \in \mathcal{P}(\mathcal{E})) \wedge \forall x((x \in y) \to \exists z((z \in x) \wedge (z \in s) \wedge \\ &\qquad \forall w(w \in \mathcal{E} \to ((w \in s) \wedge (w \in x) \leftrightarrow (w = z))))) \wedge \\ &\qquad \forall u(u \in \mathcal{E} \to ((u \in s) \leftrightarrow \exists x_1((x_1 \in y) \wedge (u \in x_1)))))) \end{aligned} \tag{7.2.1}$$

For each $A \in F(\widetilde{\mathcal{T}})$, let S_A be the set of all such choice sets generated by the predicate that follows the first $\to$ formed from (7.2.1) by deleting the $\exists s$ and letting $y = A$. Of course, this set exists within our set theory. Now let $\mathcal{C} = \{S_A \mid A \in F(\widetilde{\mathcal{T}})\}$.

Consider ${}^*\mathcal{C}$ and ${}^*(S_A)$. Then $s \in {}^*(S_A)$ iff s satisfies (7.2.1) as interpreted in ${}^*\mathcal{M}$. Since we are working in an enlargement, there exists an internal $Q \in {}^*(F(\widetilde{\mathcal{T}}))$ such that ${}^\sigma\widetilde{\mathcal{T}} \subset Q \subset {}^*\widetilde{\mathcal{T}}$. Recall that ${}^\sigma\widetilde{\mathcal{T}} = \{{}^*\mathbf{T} \mid \mathbf{T} \in \widetilde{\mathcal{T}}\}$. Also ${}^\sigma\mathbf{T} \subset {}^*\mathbf{T}$ for each $\mathbf{T} \in \widetilde{\mathcal{T}}$. From the definition of ${}^*\mathcal{C}$, there is an internal set S_Q and $s \in S_Q$ iff s satisfies the internal defining predicate for members of S_Q and this set is the set of all such s. ($\Rightarrow$) Consequently, since for each $\mathbf{T} \in \widetilde{\mathcal{T}},\ {}^*\mathbf{T} \in Q$, then the generally external $s' = \{s \cap {}^*\mathbf{T} \mid \mathbf{T} \in \widetilde{\mathcal{T}}\}$ satisfies the $\Rightarrow$ for (i). Note, however, that for ${}^*\mathbf{T},\ \mathbf{T} \in \widetilde{\mathcal{T}}$, it is possible that $s \cap {}^*\mathbf{T} = \{{}^*[f]\}$ and ${}^*[f] \in {}^\sigma\mathbf{T}$. In this case, by the finiteness of $[f]$ it follows that $[f] = {}^*[f]$ implies that $s \cap {}^*\mathbf{T} = \{[f]\}$. Now let $\mathcal{S} = \{s' \mid s \in S_Q\}$. In general, $\mathcal{S}$ is an external object.

(⇐) Consider the internal set S_Q. Let s' be the set as defined by the right-hand side of (i). For each internal $x \in s'$ and applying, if necessary, the *-axiom of choice for *-finite sets, we have the internal set $A_x = \{y \mid (y \in S_Q) \wedge (x \in y)\}$ is nonempty. The set $\{A_x \mid x \in s'\}$ has the finite intersection property. For, let nonempty internal $B = \{x_1, \ldots, x_n\}$. Then the set $A_B = \{y \mid (y \in S_Q) \wedge (x_1 \in y) \cdots \wedge (x_n \in y)\}$ is internal and nonempty by the *-axiom of choice for *-finite sets. Since we are in an enlargement and s' is countable, then $D = \bigcap\{A_x \mid x \in s'\} \neq \emptyset$. Now take any $s \in D$. Then $s \in S_Q$ and from the definition of $\mathcal{S}$, $s' \in \mathcal{S}$. This completes the proof.▮

[Note: Theorem 7.2.1 may be used to model physical developmental paradigms associated with event sequences.]

Although it is not necessary, for this particular investigation, the set $\mathcal{S}$ may be considered *a set of all developmental paradigms.* Apparently, $\mathcal{S}$ contains every possible developmental paradigm for all possible frozen segments and $\mathcal{S}$ contains paradigms for any *-totality $^*\mathbf{T}$. There are *-frozen segments contained in various s' that can be assumed to be unreadable sentences since $^\sigma\mathbf{T} \neq {}^*\mathbf{T}$.]]

Let $A \in F(\widetilde{\mathcal{T}})$ and $M(A)$ be a subset of S_A for which there exists a written set of rules that selects some specific member of S_A. Obviously, this may be modeled by means of functional relations. First, $M(A) \subset S_A$ and it follows, from the difference in cardinalities, that there are infinitely many members of $^*(S_A)$ for which there does not exist a readable rule that will select such members. However, this does not preclude the possibility that there is a set of purely unreadable sentences that do determine a specific member of $^*S_A - {}^\sigma M(A)$. This might come about in the following manner. Suppose that H is an infinite set of formal sentences that is interpreted to be a set of rules for the selection of distinct members of $M(A)$. Suppose we have a bijection $h\colon M(A) \to \mathbf{H}$ that represents this selection process. Let $^*\mathcal{M}$ be at least a polysaturated enlargement of $\mathcal{M}$, and consider $^\sigma f : {}^\sigma(M(A)) \to {}^\sigma\mathbf{H}$. The map $^\sigma f$ is also a bijection and $^\sigma f : {}^\sigma(M(A)) \to {}^*\mathbf{H}$. Since $|^\sigma(M(A))| < |\mathcal{M}|$, it is well-known that there exists an internal map $h\colon A' \to {}^*\mathbf{H}$ such that $h \mid {}^\sigma(M(A)) = {}^\sigma f$, and A', $h[A']$ are internal. Further, for internal $A' \cap {}^*(S_A) = B$, ${}^\sigma(M(A)) \subset B$. However, ${}^\sigma(M(A))$ is external. This yields that h is defined on B and $B \cap ({}^*S_A - {}^\sigma(M(A))) \neq \emptyset$. Also, ${}^\sigma\mathbf{H} \subset h[B] \subset {}^*\mathbf{H}$ implies, since $h[B]$ is internal, that ${}^\sigma H \neq h[B]$. Consequently, in this case, $h[B]$ may be interpreted as a set of *-rules that determine the selection of members of B. That is to say that there is some $[g] \in h[B] - {}^\sigma H$ and a $[k] \in {}^*S_A - {}^\sigma(M(A))$ such that $([k],[g]) \in h$. As it will be shown in the next section, the set H can be so constructed that if $[g] \in h[B] - {}^\sigma H$, then $[g]$ is unreadable.

7.3 Ultrawords

Ordinary propositional logic is not compatible with deductive quantum logic, intuitionistic logic, among others. In this section, a subsystem of propositional logic is investigated which rectifies this incompatibility. I remark that when a standard propositional language L or an informal language P isomorphic to L is considered, it will always be the case that the L or P is minimal relative to its applications. This signifies that if L or P is employed in our investigation for a developmental paradigm, then L or P is constructed only from those distinct propositional atoms that correspond to distinct members of d, etc. The same minimizing process is always assumed for the following constructions.

Let B be a formal or, informal nonempty set of propositions. Construct the language P_0 in the usual manner from B (with superfluous parentheses removed) so that P_0 forms the smallest set of formulas that contains B and such that P_0 is closed under the two binary operations $\wedge$ and $\to$ as they are formally or informally expressed. Of course, this language may be constructed inductively or by letting P_0 be the intersection of all collections of such formula closed under $\wedge$ and $\to$.

We now define the deductive system S. Assume substitutivity, parenthesis reduction and the like. Let $d = \{F_i \mid i \in \mathbb{N}\} = B$ be a development paradigm, where each F_i is a readable frozen segment and describes the behavior of a Natural-system over a time subinterval. Let the set of axioms be the schemata

$$(\mathcal{A} \wedge \mathcal{B}) \to \mathcal{A},\ \mathcal{A} \in B \tag{1}$$

$$(\mathcal{A} \wedge \mathcal{B}) \to \mathcal{B} \tag{2}$$

$$\mathcal{A} \wedge (\mathcal{B} \wedge \mathcal{C}) \to (\mathcal{A} \wedge \mathcal{B}) \wedge \mathcal{C}, \tag{3}$$

$$(\mathcal{A} \wedge \mathcal{B}) \wedge \mathcal{C} \to \mathcal{A} \wedge (\mathcal{B} \wedge \mathcal{C}). \tag{4}$$

If P_0 is considered as informal, which appears to be necessary for some applications, where the parentheses are replaced by the concept of symbol strings being to the "left" or "right" of other symbol strings and the concept of strengths of connectives is used (i.e. $A \wedge B \to C$ means $((A \wedge B) \to C)$, then axioms $3 - 4$ and the parentheses in (1) and (2) may be omitted. The one rule of inference is Modus Ponens (MP). Proofs or demonstrations from hypotheses Γ contain finitely many steps, hypotheses may be inserted as steps and the last step in the proof is either a theorem if $\Gamma = \emptyset$ or if $\Gamma \neq \emptyset$, then the last step is a consequence of (a deduction from) Γ. Notice that repeated application of (4) along with (MP) will allow all left parentheses to be shifted to the right with the exception of the (suppressed) outermost left one. Thus this leads to the concept of left to right ordering of a formula. This allows for the suppression of such parentheses. In all the following, this suppression will be done and replaced with formula left to right ordering.

For each $\Gamma \subset P_0$, let $S(\Gamma)$ denote the set of all formal theorems and consequences obtained from the above defined system S. Since hypotheses may be inserted, for each $\Gamma \subset P_0,\ \Gamma \subset S(\Gamma) \subset P_0$. This implies that $S(\Gamma) \subset S(S(\Gamma))$. So, let $A \in S(S(\Gamma))$. The general concept of combining together finitely many steps from various proofs to yield another formal proof leads to the result that $A \in S(\Gamma)$. Therefore, $S(\Gamma) = S(S(\Gamma))$. Finally, the finite step requirement also yields the result that if $A \in S(\Gamma)$, then there exists a finite $F \subset \Gamma$ such that $A \in S(F)$. Consequently, S is a finitary consequence operator and observe that if C is the propositional consequence operator, then $S(\Gamma) \underset{\neq}{\subset} C(\Gamma)$. Of course, we may now apply the nonstandard theory of consequence operators to S.

It is well-known that the axiom schemata chosen for S are theorems in intuitionistic logic. Now consider quantum logic with the Mittelstaedt conditional $i_1(A, B) = A^{\perp} \vee (A \wedge B)$. [1] Notice that $i_1(A \wedge B, B) = (A \wedge B)^{\perp} \vee ((A \wedge B) \wedge B) = (A \wedge B)^{\perp} \vee (A \wedge B) = I$ (the upper unit.) Then $i_1((A \wedge B), A) = (A \wedge B)^{\perp} \vee ((A \wedge B) \wedge A) = (A \wedge B)^{\perp} \vee (A \wedge B) = I;\ i_1((A \wedge B) \wedge C, A \wedge (B \wedge C)) = ((A \wedge B) \wedge C)^{\perp} \vee (A \wedge (B \wedge C)) = I = i_1(A \wedge (B \wedge C), (A \wedge B) \wedge C)$. Thus with respect to the interpretation of $\mathcal{A} \to \mathcal{B}$ as conditional i_1 the axiom schemata for the system S are theorems and the system S is compatible with deductive quantum logic under the Mittelstaedt conditional.

In what follows, assume $\wedge$ is interpreted as |||and|||. Recall that $d = \{F_i \mid i \in \mathbb{N}\} = B$ is a development paradigm, where each F_i is a readable frozen segment and describes the behavior of a Natural-system over a time subinterval. Although it is sufficient in most cases to consider a formal language, it is more convenient to employ an isomorphic informal language. This is especially necessary if one wishes to more closely analyze certain special subtle objects. Let $M_0 = d$. Define $M_1 = \{F_0|||\text{and}|||F_1\}$. Assume that M_n is defined. Define $M_{n+1} = \{x|||\text{and}|||F_{n+1} \mid x \in M_n\}$. From the fact that d is a developmental paradigm, where the last two symbols in each member of d is the time indicator "i.", it follows that no member of d is a member of M_n for $n > 0$. Now let $M_d = \bigcup\{M_n \mid n \in \mathbb{N}\}$. Intuitively, |||and||| behaves as a conjunction and each F_i as an atom within our language. Notice the important formal demonstration fact that for an hypothesis consisting of any member of M_n, $n > 0$, repeated applications of (1), (MP), (2), (MP) will lead to the members of d appearing in the proper time ordering at increasing (formal) demonstration step numbers.

The next theorem shows the existence of the most basic ultraword w. The ${}^*\mathbf{S}$ is the basic compatible ultralogic. This ultraword and the corresponding ultralogic generate a selected Natural event sequence along with numerous ultranatural events.

Theorem 7.3.1 *For* $\mathbf{d} = \{\mathbf{F}_i \mid i \in \mathbb{N}\}$, *there exists an ultraword* $w \in {}^*\mathbf{M_d} - {}^*\mathbf{d}$ *such that* $\mathbf{F}_i \in {}^*\mathbf{S}(\{w\})$ *(i.e.* $w\ {}^*\vdash_S \mathbf{F}_i$*) for each* $i \in \mathbb{N}$.

Proof. Consider the binary relation $G = \{(x,y) \mid (x \in \mathbf{d}) \land (y \in \mathbf{M_d} - \mathbf{d}) \land (x \in \mathbf{S}(\{y\})\}$. Suppose that $\{(x_1,y_1), \ldots (x_n,y_n)\} \subset G$. For each $i = 1, \ldots, n$ there is a unique $k_i \in \mathbb{N}$ such that $x_i = \mathbf{F}_{k_i}$. Let $m = \max\{k_i \mid (x_i = \mathbf{F}_{k_i}) \land (i = 1, \ldots, n)\}$. Let $b \in \mathbf{M}_{m+1}$. It follows immediately that $x_i \in \mathbf{S}(\{b\})$ for each $i = 1, \ldots, n$ and, from the construction of d, $b \notin \mathbf{d}$. Thus $\{(x_1,b), \ldots, (x_n,b)\} \subset G$. Consequently, G is a concurrent relation. Hence, there exists some $w \in {}^*\mathbf{M_d} - {}^*\mathbf{d}$ such that ${}^\sigma\mathbf{F}_i = \mathbf{F}_i \in {}^*\mathbf{S}(\{w\})$ for each $i \in \mathbb{N}$. This completes the proof. [See note 3.]∎

Observe that w in Theorem 7.3.1 has all of the formally expressible properties of a readable word. For example, w has a hyperfinite length, among other properties. However, since d is a denumerable set, each ultraword has a very special property.

Recall that for each $[g] \in \mathcal{E}$ there exists a unique $m \in \mathbb{N}$ and $f' \in T^m$ such that $[f'] = [g]$ and for each k such that $m < k \in \mathbb{N}$, there does not exist $g' \in T^k$ such that $[g'] = [g]$. The function $f' \in T^m$ determines all of the alphabet symbols, the symbol used for the blank space, and the like, and determines there position within the intuitive word being represented by $[g]$. Also for each j such that $0 \leq j \leq m$, $f'(j) = i(\mathrm{a}) \in i[\mathcal{W}] = T$, where $i(\mathrm{a})$ is the "encoding" in $i[\mathcal{W}]$ of the symbol "a". For each $m \in \mathbb{N}$, let $P_m = \{f \mid (f \in T^m) \land (\exists z((z \in \mathcal{E}) \land (f \in z) \land \forall x((x \in \mathbb{N}) \land (x > m) \to \neg\exists y((y \in T^x) \land (y \in z))))))\}$. An element $n \in {}^*\mathcal{W}$ is a *subtle alphabet symbol* if there exists $m \in \mathbb{N}$ and $f \in {}^*(P_m)$, or if $m \in \mathbb{N}_\infty$ and $f \in P_m$, and some $j \in {}^*\mathbb{N}$ such that $f(j) = n$. A symbol is a *pure subtle alphabet symbol* if $f(j) = n \notin i[\mathcal{W}]$. Subtle alphabet symbols can be characterized in ${}^*\mathcal{E}$ for they are singleton objects. A $[g] \in {}^*\mathcal{E}$ represents a subtle alphabet symbol iff there exists some $f \in ({}^*T)^0$ such that $[f] = [g] = [(0, f(0))]$, $f = \{(0, f(0))\}$.

Theorem 7.3.2 *Let* $\mathrm{d} = \{\mathrm{F}_i \mid i \in \mathbb{N}\}$ *be a denumerable developmental paradigm. For an ultraword* $w = [g] \in {}^*\mathcal{E}$ *that exists by Theorem 7.3.1, there exists* $\delta \in \mathbb{N}_\infty$ *and infinitely many* $\nu \in \mathbb{N}_\infty$ *such that* $[f] = [g]$, $f \in P_\delta$ *and each* $f(\nu) \in {}^*(i[\mathrm{P}_0]) - (i[\mathrm{P}_0])$ *is a pure subtle alphabet symbol.*

Proof. Since $\mathbf{d}$ is denumerable, consider a bijection $h\colon \mathbb{N} \to \mathbf{d}$ such that $h(n) = \mathbf{F}_n = [(0,q_n)]$, $q_n \in i[\mathrm{d}] \subset i[\mathrm{P}_0]$. From the definition of $\mathbf{M_d} - \mathbf{d}$, if $[g] \in \mathbf{M_d} - \mathbf{d}$, there exists a unique $m,\ n \in \mathbb{N}$ $(n \geq 1)$ and $f' \in (i[\mathrm{P}_0])^m$ such that $h[[0,n]] \subset \mathbf{S}(\{[g]\})$ and **(0)** $[f'] = [g]$, and **(1)** $m = 2n$, **(2)** for each even $2k \leq m$, $(k \geq 0)$, $f'(2k) = q_k \in i([\mathrm{P}_0]) \subset i[\mathcal{W}]$, $[(0,q_k)] \in \mathbf{d}$. All such q_k are distinct and $[(0,q_k)] \in \mathbf{S}(\{f'\})$. For each odd $2k+1 \leq m$, $f'(2k+1) = i(|||\mathrm{and}|||)$. **(3)** For each $h(k) \in h[[0,n]]$ there exist distinct even $2k \leq m$ such that $f'(2k) = q_k \in i[\mathrm{P}_0] \subset i\mathcal{W}]$ and conversely, and $h(k) = [(0,q_k)] \in \mathbf{d}$ all such q_k being distinct. Also note that for each $i \in \mathbb{N}$, $h(i) \in \mathbf{S}(\{[g]\})$ iff $h(i) \in h[[0,n]]$. **(4)** There exists a unique $r \in \mathbb{N}$ such that $r > m$ and an $f" \in P_r$ such that $[f"] = [g]$. **(5)** For each of the $n+1$ distinct k's that exist from the first part of **(2)**, there exists at least $n+1$ distinct $r_k \in \mathbb{N}$ such that $0 \leq r_n \leq r$ and $n+1$ distinct $f"(r_k) = i(\mathrm{b}_k)$, where b_k is a symbol representing the natural number that appears as the next to the last symbol in a member of d.

Let $w = [g] \in {}^*\mathbf{M} - {}^*\mathbf{d}$ be the ultraword that exists by Theorem 7.3.1. Then there exists some $\delta \in {}^*\mathbb{N}$ and a unique $\nu \in {}^*\mathbb{N}$ as well as $f' \in ({}^*\mathbf{i}[\mathbf{P_0}])^\delta$ such that ${}^*h[[0,\nu]] \subset {}^*\mathbf{S}(\{[f']\}) = {}^*\mathbf{S}(\{[g]\})$ with the *-transfer of properties (**1**) — (**5**). From Theorem 7.3.1, ${}^\sigma(h[\mathbb{N}]) \subset {}^*\mathbf{S}(\{[f']\})$ yields, by application, of (**3**) that $h[\mathbb{N}] = {}^\sigma(h[\mathbb{N}]) \subset {}^*h[[0,\nu]]$. This implies that ${}^*h[[0,\nu]]$ is infinite and internal. From (**1**), we obtain that $\delta \in \mathbb{N}_\infty$. From (**4**), there exists a unique $\rho > \delta$, hence $\rho \in \mathbb{N}_\infty$, and a unique $f \in P_\delta$ such that $[f] = [g] = [f']$. From (**5**), there exist at least ν *-distinct, and hence distinct, $\rho_k \in {}^*\mathbb{N}$ such that the ν distinct $f(\rho_k) \in {}^*(i[\mathrm{P}_0])$. Since ${}^*\mathcal{M}$ is an ultrapower or ultralimit enlargement based upon $\mathbb{N}$, it follows that $|[0,\nu]| \geq 2^{\aleph_0}$. Consequently, the cardinality of the ν distinct $f(\rho_k)$ is greater than or equal to $2^{\aleph_0}$. Since $i[\mathcal{W}]$ is denumerable, the cardinality of the set of purely subtle alphabet symbols contained in the set of ν distinct $f(\rho_k) \geq 2^{\aleph_0}$. This complete the proof. ▮

With respect to the proof of Theorem 7.3.2, the function f determines the alphabet composition of the ultraword w. The word w is unreadable not only due to its infinite length but also due to the fact that it is composed of infinitely many purely subtle alphabet symbols.

The developmental paradigm d utilized for the two previous theorems is composed entirely of readable sentences. We now investigate what happens if a developmental paradigm contains countably many unreadable sentences. Let the nonempty developmental paradigm d' be composed of at most countably many members of ${}^*\mathcal{E} - \mathcal{E}$ and let $d' \subset {}^*\mathbf{B} \subset {}^*\mathbf{P}_0$. Construct, as previously, the set $\mathrm{M_B}$ from B, rather than from d and suppose that $\mathrm{B} \cap \mathrm{M}_i = \emptyset,\ i \neq 0$. [This last requirement for B can be achieved as follows: construct a special symbol not originally in $\mathcal{A}$. Then this symbol along with $\mathcal{A}$ is considered the alphabet. Next only consider a B that does not contain this special symbol within any of its members. Now use this special symbol consistently with or without the spacing symbol to construct $\mathrm{M}_i\ i \neq 0$. Of course $\wedge$ is interpreted as this special symbol with or without the spacing symbol in the axiom system S.]

The w that exists by the next theorem generates, upon applying the ultralogic ${}^*\mathbf{S}$, any selected event sequence that contains not only previously selected Natural events but also selected UN-events.

Theorem 7.3.3 *Let $d' = \{[g_i] \mid i \in \mathbb{N}\}$. Then there exists an ultraword $w \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ such that for each $i \in \mathbb{N}$, $[g_i] \in {}^*\mathbf{S}(\{w\})$.*

Proof. Consider the internal binary relation $G = \{(x,y) \mid (x \in {}^*\mathbf{B}) \wedge (y \in {}^*\mathbf{M_B} - {}^*\mathbf{B}) \wedge (x \in {}^*\mathbf{S}(\{y\})\}$. Note that members of d' are members of ${}^\sigma\mathcal{E}$ or, at the most, denumerably many members of ${}^*\mathcal{E} - {}^\sigma\mathcal{E}$. From the analysis in the proof of Theorem 7.3.1, for a finite $\mathrm{F} \subset \mathrm{B}$, there exists some $\mathrm{y} \in \mathrm{M_B} - \mathrm{B}$ such that $\mathrm{F} \subset S(\{\mathrm{y}\})$. It follows by *-transfer that if F is a finite or *-finite subset of ${}^*\mathbf{B}$, then there exists some $y \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ such that $F \subset {}^*\mathbf{S}(\{y\})$. As in the proof of Theorem 7.3.1, this yields that G is at least concurrent on ${}^*\mathbf{B}$. However, $d' \subset {}^*\mathbf{B}$ and $|d'| \leq \aleph_0$. From $\aleph_1$-saturation, there exists some $w \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ such that for each $[g_i] \in d',\ [g_i] \in {}^*\mathbf{S}(\{w\})$. This completes the proof. ▮

Let $\emptyset \neq \lambda \subset \mathbb{N}$ and $\mathcal{D}_j = \{d_{ij} \mid i \in \lambda\}$ and each $d_i \subset {}^*\mathbf{B}$ is considered to be a developmental paradigm either of type d or type d' and $\mathrm{B} \cap \mathrm{M}_i = \emptyset,\ i \neq 0$. Notice that $\mathcal{D}_j$ may be either a finite or denumerable set and Theorem 7.3.1 holds for the case that $\mathrm{d} \subset \mathrm{B}$, where $w \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$. For each $d_{ij} \in \mathcal{D}_j$, use the Axiom of Choice to select an ultraword $w_{ij} \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ that exists by Theorems 7.3.1 (extended) and 7.3.3. Let $\{w_{ij} \mid i \in \lambda\}$ be such a set of ultrawords.

This next theorem shows the existence of an ultimate ultraword that by application of the ultralogic yields all of the other needed ultrawords. Thus w'_i generates, after application of the ultralogic ${}^*\mathbf{S}$, a Natural-system's event sequence, whether it contains only Natural events or selected UN-events. Then w' and ${}^*\mathbf{S}(\{w'\})$ solve Wheeler's "general grand unification problem."

Theorem 7.3.4 *There exists an ultimate ultraword* $w'_j \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ *such that for each* $i \in \lambda,\ w_{ij} \in {}^*\mathbf{S}(\{w'_j\})$ *and, hence, for each* $d_{ij} \in \mathcal{D}_j,\ d_{ij} \subset {}^*\mathbf{S}(\{w_{ij}\}) \subset {}^*\mathbf{S}(\{w'_j\})$.

Proof. For each finite $\{\mathrm{F}_1, \ldots, \mathrm{F}_n\} \subset \mathrm{M_B} - \mathrm{B}$ there is a natural number, say m, such that for $i = 1, \ldots, n,\ \mathrm{F}_i \in \mathrm{M}_j$ for some $j \leq m$. Hence, taking $\mathrm{b} \in \mathrm{M}_{m+1}$, we obtain that each $\mathrm{F}_i \in S(\{\mathrm{b}\})$. Observe that $\mathrm{b} \notin \mathrm{B}$. By *-transfer, it follows that the internal relation $G = \{(x, y) \mid (x \in {}^*\mathbf{M_B} - {}^*\mathbf{B}) \wedge (y \in {}^*\mathbf{M_B} - {}^*\mathbf{B}) \wedge (x \in {}^*\mathbf{S}(\{y\})\}$ is concurrent on internal ${}^*\mathbf{M_B} - {}^*\mathbf{B}$ and $\{w_{ij} \mid i \in \lambda\} \subset {}^*\mathbf{M_B} - {}^*\mathbf{B}$. Again $\aleph_1$-saturation yields that there is some $w'_j \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ such that for each $i \in \lambda,\ w_{ij} \in {}^*\mathbf{S}(\{w'_j\})$. The last property is obtained from $d_{ij} \subset {}^*\mathbf{S}(\{w_{ij}\}) \subset {}^*\mathbf{S}({}^*\mathbf{S}(\{w'_j\})) = {}^*\mathbf{S}(\{w'_j\})$ since $\{w_i\}$ is an internal subset of ${}^*\mathbf{P}_0$. This completes the proof.∎

Corollary 7.3.4.1 *Let* $\emptyset \neq \gamma \subset \mathbb{N}$. *There exists an ultimate ultraword* $w' \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ *such that for each* $j \in \gamma,\ w'_j \in {}^*\mathbf{S}(\{w'\})$ *and, hence, for each* $d_{ij} \in \bigcup \mathcal{D}_j,\ d_{ij} \subset {}^*\mathbf{S}(\{w'_j\}) \subset {}^*\mathbf{S}(\{w'\})$.

The same analysis used to obtain Theorem 7.3.2 can be applied to the ultrawords of Theorems 7.3.3 and 7.3.4.

7.4 Ultracontinuous Deduction

In 1968, a special topology on the set of all nonempty subsets of a given set X was constructed and investigated by your author. We apply a similar topology to subsets of $\mathcal{E}$.

Suppose that nonempty $X \subset \mathcal{E}$. Let τ be the discrete topology on X. In order to topologize $\mathcal{P}(X)$, proceed as follows: for each $G \in \tau$, let $N(G) = \{A \mid (A \subset X) \wedge (A \subset G)\} = \mathcal{P}(G)$. Consider $\mathcal{B} = \{N(G) \mid G \in \tau\}$ to be a base for a topology τ_1 on $\mathcal{P}(X)$. Let $A \in N(G_1) \cap N(G_1)$. The discrete topology implies that $N(A)$ is a base element and that $N(A) \subset N(G_1) \cap N(G_2)$. There is only one member of $\mathcal{B}$ that contains X and this is $\mathcal{P}(X)$. Thus if $\mathcal{P}(X)$ is covered by members of $\mathcal{B}$, then $N(X) = \mathcal{P}(X)$ is one of these covering objects. Thus $(\mathcal{P}(X), \tau_1)$ is a compact space. Further, since $N(\emptyset) \subset N(G)$ for each $G \in \tau$, the space $(\mathcal{P}(X), \tau_1)$ is connected. The topology τ_1 is a special case of a more general topology with the same properties. [2] Suppose that $D \subset X$. Let $D \in N(G) = \mathcal{P}(G), G \in \tau$. Then $D \in N(D) \subset N(G)$. This yields that the nonstandard monad is $\mu(D) = \bigcap\{{}^*N(G) \mid N(G) \in \mathcal{B}\} = {}^*(\mathcal{P}(D)) = {}^*\mathcal{P}({}^*D)$.

This next theorem shows that the ultralogic ${}^*\mathbf{S}$ produces Natural and ultranatural events in an ultracontinuous manner.

Theorem 7.4.1 *Any consequence operator* $C \colon (\mathcal{P}(X), \tau_1) \to (\mathcal{P}(X), \tau_1)$ *is continuous.*

Proof. Let $A \in \mathcal{P}(X)$ and $H \in {}^*\mathbf{C}[\mu(A)]$. Then there exists some $B \in \mu(A)$ such that ${}^*\mathbf{C}(B) = H$. Hence, $B \in {}^*\mathcal{P}({}^*A)$. By *-transfer of a basic property of our consequence operators, ${}^*\mathbf{C}(B) \subset {}^*\mathbf{C}({}^*A) = {}^*(\mathbf{C}(A))$. Thus ${}^*(\mathbf{C}(B)) \in {}^*(\mathcal{P}(\mathbf{C}(A)))$ implies that ${}^*\mathbf{C}(B) \in \mu(A)$. Therefore, ${}^*\mathbf{C}[\mu(A)] \subset \mu([\mathbf{C}(A)])$. Consequently, $\mathbf{C}$ is continuous. ∎

Corollary 7.4.1.1 *For any* $X \subset \mathcal{E}$, *and any consequence operator* $\mathbf{C} \colon \mathcal{P}(X) \to \mathcal{P}(X)$, *the map* ${}^*\mathbf{C} \colon {}^*(\mathcal{P}(X)) \to {}^*(\mathcal{P}(X))$ *is ultracontinuous.*

Corollary 7.4.1.2 *Let* d [*resp.* d', d *or* d'] *be a developmental paradigm as defined for Theorem 7.3.1* [*resp. Theorem 7.3.3, 7.3.4*]. *Let* w *be a ultraword that exists by Theorem 7.3.1* [*resp Theorem 7.3.3, 7.3.4*]. *Then* d [*resp.* d', d *or* d'] *is obtained by means of a ultracontinuous subtle deductive process applied to* $\{w\}$.

Recall that in the real valued case, a function $f\colon [a,b] \to \mathbb{R}$ is uniformly continuous on $[a,b]$ iff for each $p,\ q \in {}^*[a,b]$ such that $p-q \in \mu(0)$, then $f(p)-f(q) \in \mu(0)$. If $D \subset [a,b]$ is compact, then $p,\ q \in {}^*D$ and $p-q \in \mu(0)$ imply that there is a standard $r \in D$ such that $p,\ q \in \mu(r)$. Also, for each $r \in D$ and any $p,\ q \in \mu(r)$, it follows that $p-q \in \mu(r)$. Thus, if compact $D \subset [a,b]$, then $f : D \to \mathbb{R}$ is uniformly continuous iff for every $r \in D$ and each $p,\ q \in \mu(r),\ \ {}^*f(p),\ \ {}^*f(q) \in \mu(f(r))$. With this characterization in mind, it is clear that any consequence operator $\mathbf{C}\colon \mathcal{P}(X) \to \mathcal{P}(X)$ satisfies the following statement. For each $A \in \mathcal{P}(X)$ and each $p,\ q \in \mu(A),\ \ {}^*\mathbf{C}(p),\ \ {}^*\mathbf{C}(q) \in \mu(\mathbf{C}(A))$.

From the above discussion, one can think of ultracontinuity as being a type of ultrauniform continuity.

7.5 Hypercontinuous Gluing

There are various methods that can be used to investigate the behavior of adjacent frozen segments. All of these methods depend upon a significant result relative to discrete real or vector valued functions. The major goal in this section is to present a complete proof of this major result and to indicate how it is applied.

First, as our standard structure, consider either the intuitive real numbers as atoms or axiomatically a standard structure with atoms $\mathbf{ZFR} = \mathbf{ZF} + \mathbf{AC} + \mathcal{W}(\text{atoms}) + A(\text{atoms}) + |A| = \text{c}$, where A is isomorphic to the real numbers and $\mathcal{W} \cap A = \emptyset$. Then, as done previously, there is a model $\langle C, \in, = \rangle$ within our $\mathbf{ZF} + \mathbf{AC}$ model for $\mathbf{ZFR}$, where A has all of the ordered field properties as the real numbers. A superstructure $\langle \mathcal{R}, \in, = \rangle$ is constructed in the usual manner, where the superstructure $\langle \mathcal{N}, \in, = \rangle$ is a substructure. Proceeding as in Chapter 2, construct ${}^*\mathcal{M}_1 = \langle {}^*\mathcal{R}, \in = \rangle$ and $\mathcal{Y}_1$. The structure $\mathcal{Y}_1$ is called the *Extended Grundlegend Structure* — the EGS. The Grundlegend Structure is a substructure of $\mathcal{Y}_1$.

It is important to realized in what follows that the objects utilized for the G-structure *interpretations* are nonempty finite equivalence classes of partial sequences. Due to this fact, the following results should not lead to ambiguous interpretations.

As a preliminary to the technical aspects of this final section, we introduce the following definition. A function $f\colon [a,b] \to \mathbb{R}^m$ is *differentiable-C* on $[a,b]$ if it is continuously differentiable on (a,b) except at finitely many removable discontinuities. This definition is extended to the end points $\{a,b\}$ by application of one-sided derivatives. For any $[a,b]$, consider a partition $P = \{a_0, a_1, \cdots, a_n, a_{n+1}\},\ n \geq 1,\ a = a_0,\ b = a_{n+1}$ and $a_{j-1} < a_j,\ 1 \leq j \leq n+1$. For any such partition P, let the real valued function g be defined on the set $D = [a_0, a_1) \cup (a_1, a_2) \cup \cdots \cup (a_n, a_{n+1}]$ as follows: for each $x \in [a_0, a_1)$, let $g(x) = r_1 \in \mathbb{R}$; for each $x \in (a_{j-1}, a_j)$, let $g(x) = r_j \in \mathbb{R},\ 1 < i \leq n$; for each $x \in (a_n, b]$, let $g(x) = r_{n+1} \in \mathbb{R}$. It is obvious that g is a type of simple step function. Notationally, let $\mathcal{F}(A,B)$ denote the set of all functions with domain A and codomain B.

Theorem 7.5.1. shows that if a time fracture occurs of the minimum or intermediate type, then there exists an ultracontinuous, ultrauniform and ultrasmooth alteration process within the NSP-world that yields all of the alterations.

Theorem 7.5.1 *There exists a function* $G \in {}^*(\mathcal{F}([a,b],\mathbb{R}))$ *with the following properties.*

(i) The function G *is *-continuously *-differentiable and *-uniformly *-continuous on* ${}^*[a,b]$,

(ii) for each odd $n \in {}^*\mathbb{N},\ (n \geq 3),\ G$ *is *- differentiable-C of order* n *on* ${}^*[a,b]$,

(iii) for each even $n \in {}^*\mathbb{N},\ G$ *is *-continuously *-differentiable in* ${}^*[a,b]$ *except at finitely many points,*

(iv) if $c = \min\{r_1,\cdots,r_{n+1}\},\ d = \max\{r_1,\cdots,r_{n+1}\}$, *then the range of* $G = {}^*[c,d],\ \mathtt{st}(G)$ *at least maps* D *into* $[c,d]$ *and* $(\mathtt{st}(G))|D = g$.

Proof. First, for any real c, d, where $d \neq 0$, consider the finite set of functions

$$h_j(x,c,d) = (1/2)(r_{j+1} - r_j)\Big(\sin\big((x-c)\pi/(2d)\big) + 1\Big) + r_j, \tag{7.5.1}$$

$1 \leq j \leq n$. Each h_j is continuously differentiable for any order at each $x \in \mathbb{R}$. Observe that for each odd $m \in \mathbb{N}$, each m'th derivative $h_j^{(m)}$ is continuous at $(c+d)$ and $(c-d)$ and $h_j^{(m)}(c+d) = h_j^{(m)}(c-d) = 0$ for each j.

Let positive $\delta \in \mu(0)$. Consider the finite set of internal intervals $\{[a_0, a_1 - \delta), (a_1 + \delta, a_2 - \delta), \cdots, (a_n + \delta, b]\}$ obtained from the partition P. Denote these intervals in the expressed order by $I_j,\ 1 \leq j \leq n+1$. Define the internal function

$$G_1 = \{(x, r_1)|x \in I_1\} \cup \cdots \cup \{(x, r_{n+1})|x \in I_{n+1}\}. \tag{7.5.2}$$

Let internal $I_j^\dagger = [a_j - \delta, a_j + \delta],\ 1 \leq j \leq n$, and for each $x \in I_j^\dagger$, let internal

$$G_j(x) = (1/2)(r_{j+1} - r_j)\Big({}^*\!\sin\big((x - a_j)\pi/(2\delta)\big) + 1\Big) + r_j. \tag{7.5.3}$$

Define the internal function

$$G_2 = \{(x, G_1(x))|x \in I_1^\dagger\} \cup \cdots \cup \{(x, G_n(x))|x \in I_n^\dagger\}. \tag{7.5.4}$$

The final step is to define $G = G_1 \cup G_2$. Then $G \in {}^*(\mathcal{F}([a,b],\mathbb{R}))$.

By *-transfer, the function G_1 has an internal *-continuous *- derivative $G_1^{(1)}$ such that $G_1^{(1)}(x) = 0$ for each $x \in I_1 \cup \cdots \cup I_{n+1}$. Applying *-transfer to the properties of the functions $h_j(x,c,d)$, it follows that G_2 has a unique internal *-continuous *-derivative

$$G_2^{(1)} = (1/(4\delta))(r_{j+1} - r_j)\pi\Big({}^*\!\cos\big((x - a_j)\pi/(2\delta)\big)\Big) \tag{7.5.5}$$

for each $x \in I_1^\dagger \cup \cdots \cup I_n^\dagger$. The results that the *-left limit for the internal $G_1^{(1)}$ and the *-right limit for internal $G_2^{(1)}$ at each $a_j - \delta$ as well as the *-left limit of $G_2^{(1)}$ and *-right limit of $G_1^{(1)}$ at each $a_j + \delta$ are equal to 0 and $0 = G_2^{(1)}(a_j - \delta) = G_2^{(1)}(a_j + \delta)$ imply that internal G has a *-continuous *- derivative $G^{(1)} = G_1^{(1)} \cup G_2^{(1)}$ defined on ${}^*[a,b]$.

A similar analysis and *-transfer yield that for each $m \in {}^*\mathbb{N},\ m \geq 2,\ G$ has an internal *-continuous *-derivative $G^{(m)}$ defined at each $x \in {}^*[a,b]$ except at the points $a_j \pm \delta$ whenever $r_{j+1} \neq r_j$. However, it is obvious from the definition of the functions h_j that for each odd $m \in {}^*\mathbb{N},\ m \geq 3$, each internal $G^{(m)}$ can be made *-continuous at each $a_j \pm \delta$ by simply defining $G^{(m)}(a_j \pm \delta) = 0$ and with this parts (i), (ii), and (iii) are established.

For part (iv), assume that $r_j \leq r_{j+1}$. From the definition of the functions h_j, it follows that for each $x \in I_j \cup I_j^\dagger \cup I_{j+1},\ r_j \leq G(x) \leq r_{j+1}$. The nonstandard intermediate value theorem implies that $G\big[\ {}^*[a_j,a_{j+1}]\big] =\ {}^*[r_j,r_{j+1}]$ and in like manner for the case that $r_{j+1} < r_j$. Hence, $G\big[\ {}^*[a,b]\big] =\ {}^*[c,d]$. Clearly, $\mathtt{st}(\ {}^*D) = [a,b]$. If $p \in D$ and $x \in \mu(p) \cap\ {}^*D$, then $G(x) = r_j = g(p)$ for some j such that $1 \leq j \leq n+1$. This completes the proof. ▮

The nonstandard approximation theorem 7.5.1 can be extended easily to functions that map D into $\mathbb{R}^m$. For example, assume that $F\colon D \to \mathbb{R}^3$, the component functions $F_1,\ F_2$ are continuously differentiable on $[a,b]$; but that F_3 is a g type step function on D. Then letting $H = (\ {}^*F_1,\ {}^*F_2, G)$, on ${}^*[a,b]$, where G is defined in Theorem 4.1, we have an internal *-continuously *-differentiable function $H\colon\ {}^*[a,b] \to {}^*\mathbb{R}^3$, with the property that $\mathtt{st}(H)|D = F$.

With respect to Theorem 7.5.1, it is interesting to note that if h_j is defined on $\mathbb{R}$, then for even orders $n \in \mathbb{N}$,

$$\text{(7.5.6)} \qquad |\ h_j^{(n)}(c \pm d)\ | = \left|\frac{(r_{j+1}-r_j)\pi^n}{2^{n+1}d^n}\right| = 0$$

for $r_{j+1} = r_j$ but not 0 otherwise. If $r_{j+1} - r_j \neq 0$, then $G_2^{(n)}(a_j \pm \delta)$ is an infinite nonstandard real number. Indeed, if m_i is an increasing sequence of even numbers in ${}^*\mathbb{N}$ and $r_{j+1} \neq r_j$, then $|G_2^{(m_i)}(a_j \pm \delta)|$ forms a decreasing sequence of nonstandard infinite numbers. The next result is obvious from the previous result.

Corollary 7.5.1.1 *For each $n \in {}^*\mathbb{N}$, then internal $G^{(n)} = G_1^{(n)} \cup G_2^{(n)}$ is *-bounded on ${}^*[a,b]$.*

Let $D(a,b)$ be the set of all bounded and piecewise continuously differentiable functions defined on $[a,b]$. By considering all of the possible (finitely many) subintervals, where $f \in D(a,b)$, it follows from the Riemann sum approach that for each real $\nu > 0$, there exists a real $\nu_1 > 0$ such that for each real $\nu_i,\ 0 < \nu_i < \nu_1$, a sequence of partitions $P_i = \{a = b_0^i < \cdots < b_{k_i}^i = b\}$ can be selected such that the mesh$(P_i) \leq \nu_i$ and

$$\text{(7.5.7)} \qquad |(f(b) - f(a)) - \sum_{n=1}^{k_i} f'(t_n)(b_n^i - b_{n-1}^i)| < \nu$$

for any $t_n \in (b_{n-1}^i, b_n^i)$.

Moreover, for any given number M, the sequence of partitions can be so constructed such that there exists a j such that for each $i > j,\ k_i > M$, where P_i and P_j are partitions within the sequence of partitions. By *-transfer of these facts and by application of Theorem 7.5.1 and its corollary we have the next result.

Corollary 7.5.1.2 *For each $n \in \mathbb{N}$ and each internal $G^{(n)}$, the difference $G^{(n)}(b) - G^{(n)}(a)$ is infinitesimally close to an (externally) infinity *-finite sum of infinitesimals.*

A developmental paradigm is a very general object and, therefore, can be used for numerous applications. At present, developmental paradigms are still being viewed from the *substratum* or external world. For what follows, it is assumed that a developmental paradigm d traces the evolutionary history of a specifically named natural system or systems. In this first application, let each $\mathrm{F}_i \in \mathrm{d}$ have the following property (**P**).

> F_i describes "the general behavior and characteristics of the named natural system S_1 as well as the behavior and characteristics of named constituents contained within S_1 at time t_i."

Recall that for $\mathrm{F}_i,\ \mathrm{F}_{i+1} \in \mathrm{d}$, there exist unique functions $f_0 \in \mathbf{F}_i = [f],\ g_0 \in \mathbf{F}_{i+1} = [g]$ such that $f_0,\ g_0 \in T^0$ and $\{(0, f_0(0))\} \in [f],\ \{(0, g_0(0))\} \in [g]$. Thus, to each $\mathrm{F}_j \in \mathrm{d}$, correspond the unique natural number $f_0(0)$. Let $D = [t_{i-1}, t_i) \cup (t_i, t_{i+1}]$ and define $f_1 \colon D \to \mathbb{N}$ as follows: for each $x \in [t_{i-1}, t_i)$, let $f_1(x) = f_0(0)$; for each $x \in (t_i, t_{i+1}]$, let $f_1(x) = g_0(0)$. Application of theorem 7.5.1 yields the internal function G such that $G|D = f_1$. For these physical applications, utilize the term "substratum" in the place of the technical terms "pure nonstandard." [Note: Of course, elsewhere, the term "pure NSP-world" or simply the "NSP-world" is used as a specific name for what has here been declared as the substratum.] This yields the following statements, where the symbols F_i and F_{i+1} are defined and characterized by the expression inside the quotation marks in property (**P**).

> (**A**): There exists a substratum hypercontinuous, hypersmooth, hyperuniform process G that binds together F_i and F_{i+1}.
>
> (**B**): There exists a substratum hypercontinuous, hypersmooth, hyperuniform alteration process G that transforms F_i into F_{i+1}.
>
> (**C**): There exists an ultracontinuous subtle force-like (i.e. deductive) process that yields F_i for each time t_i within the development of the natural system.

In order to justify (**A**) and (**B**), specific measures of physical properties associated with constituents may be coupled together. Assume that for a subword $\mathrm{r}_i \in F_i \in \mathrm{d}$, the symbols r_i denote a numerical quantity that aids in characterizing the behavior of an object in a system S_1 or the system itself. Let $(\mathbf{M}_1)$ be the statement:

> "There exists a substratum hypercontinuous, hypersmooth, hyperuniform functional process G_i such that G_i when restricted to the standard mathematical domain it is f_i and such that G_i hypercontinuously changes r_i for system S_1 at time t_i into r_{i+1} for system S_1 at time t_{i+1}."

This modeling procedure yields the following interpretation:

> (**D**) If there exists a continuous or uniform [resp. discrete] functional process f_i that changes r_i for S_1 at time t_i into r_{i+1} for S_1 at time t_{i+1}, then $(\mathbf{M}_1)$.

At a particular moment t_i, two natural systems S_1 and S_2 may interface. More generally, two very distinct developmental paradigms may exist one d_1 at times prior to t_i (in the t_i past) and one d_2 at time after t_i (in the t_i future). We might refer to the time t_i as a *standard time fracture.* Consider the developmental paradigm $\mathrm{d}_3 = \mathrm{d}_1 \cup \mathrm{d}_2$. In this case, the paradigms may be either of type d or d'. For the type d', the corresponding system need not be considered a natural system but could be a pure substratum system.

At t_i an $\mathrm{F}_i \in \mathrm{d}_3$ can be characterized by statement (**P**) (with the term natural removed if F_i is a member of a d'). In like manner, F_{i+1} at time t_{i+1} can be characterized by (**P**). Statements (**A**), (**B**), (**C**) can now be applied to d_3 and a modified statement (**D**), where the second symbol string S_1 is changed to S_2. Notice that this modeling applies to the actual human ability that only allows for two discrete descriptions to be given, one for the interval $[t_{i-1}, t_i)$ and one for the interval $(t_i, t_{i+1}]$. From the modeling viewpoint, this is often sufficient since the length of the time intervals can be made smaller than Planck time.

Recall that an analysis of the scientific method used in the investigation of natural system should take place exterior to the language used to describe the specific system development. Suppose that $\mathcal{D}$ is the language accepted for a scientific discipline and that within $\mathcal{D}$ various expressions from mathematical theories are used. Further, suppose that enough of the modern theory of sets is employed so that the EGS can be constructed. The following statement would hold true for $\mathcal{D}$.

If by application of first-order logic to a set of non-mathematical premises taken from $\mathcal{D}$ it is claimed that it is not logically possible for statements such as (**A**), (**B**), (**C**) *and* (**D**) *to hold, then the set of premises is inconsistent.*

CHAPTER 7 REFERENCES

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NOTES

[1] The actual members, $\mathrm{F_i}$, of a developmental paradigm d need not be unique. However, the specific information contained in each readable word used for a specific $i \in \mathbb{N}$ is unique.

[2] Depending upon the application, a single standard word may also be termed as an ultraword.

[3] For a new more detailed method to obtain ultrawords for a refined developmental paradigm, see pages 4 – 7 of http://arxiv.org/abs/math/0605120

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9.1 An Extension.

Although it is often not necessary, we assume when its useful that we are working within the EGS. Further, this structure is assumed to be $2^{|\mathcal{M}_1|}$-saturated, where $\mathcal{M}_1 = \langle \mathcal{N}, \in, = \rangle$ and the ground set is $\mathcal{W}' \cup \mathbb{R}$, even though such a degree of saturation can usually be reduced. Referring to the paragraph prior to Theorem 7.3.3, it can be assumed that the developmental paradigm $d' \subset {}^*\mathbf{B} \subset {}^*\mathbf{P}_0$. It is not assumed that such a developmental paradigm is obtained from the process discussed in Theorem 7.2.1, although a modification of the proof of Theorem 7.2.1 appears possible in order to allow this method of selection.

Theorem 9.1.1 *Let* $d' = \{[g_i] \mid i \in \lambda\},\ |\lambda| < 2^{|\mathcal{M}_1|}$. *There exists an ultraword* $w \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ *such that for each* $i \in \lambda,\ [g_i] \in {}^*\mathbf{S}(\{w\})$.

Proof. The same as Theorem 7.3.3 with the change in saturation.∎

Let $\mathcal{D} = \{d_i \mid i \in \lambda\},\ |\lambda| < 2^{|\mathcal{M}_1|},\ |d_i| < 2^{|\mathcal{M}_1|}$ and each $d_i \subset {}^*\mathbf{B}$ is considered to be a developmental paradigm either of type d or type d'. For each $d_i \in \mathcal{D}$, use the Axiom of Choice to select an ultraword $w_i \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ that exists by Theorems 9.1.1. Let $\{w_i \mid i \in \lambda\}$ be such a set of ultrawords.

Theorem 9.1.2 *There exists an ultraword* $w' \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ *such that for each* $i \in \lambda,\ w_i \in {}^*\mathbf{S}(\{w'\})$ *and, hence, for each* $d_i \in \mathcal{D},\ d_i \subset {}^*\mathbf{S}(\{w'\})$.

Proof. The same as Theorem 7.3.4 with the change in saturation.∎

9.2 NSP-World Alphabets.

First, recall the following definition. $P_m = \{f \mid (f \in T^m) \wedge (\exists z((z \in \mathcal{E}) \wedge (f \in z) \wedge \forall x((x \in \mathbb{N}) \wedge (x > m) \to \neg\exists y((y \in T^x) \wedge (y \in z)))))\}$. The set $T = i[\mathcal{W}]$. The set P_m determines the unique partial sequence $f \in [g] \in \mathcal{E}$ that yields, for each $j \in \mathbb{N}$ such that $0 \leq j \leq m,\ f(j) = i(\mathrm{a})$, where $i(\mathrm{a})$ is an "encoding" in $i[\mathcal{W}]$ of the alphabet symbol "a" used to construct our intuitive language $\mathcal{W}$. The set $[g]$ represents an intuitive word constructed from such an alphabet of symbols.pars Within the discipline of Mathematical Logic, it is assumed that there exists symbols — a sequence of variables — each one of which corresponds, in a one-to-one manner, to a natural number. Further, under the subject matter of generalized first-order theories [2], it is assumed that the cardinality of the set of constants is greater than $\aleph_0$. In the forthcoming investigation, it may be useful to consider an alphabet that injectively corresponds to the real numbers $\mathbb{R}$. This yields a new alphabet $\mathcal{A}'$ containing our original alphabet. A new collection of words $\mathcal{W}'$ composed of nonempty finite strings of such alphabet symbols may be constructed. It may also be useful to well-order $\mathbb{R}$. The set $\mathcal{E}$ also exists with respect to the set of words $\mathcal{W}'$. Using the ESG, many previous results in this book now hold with respect to $\mathcal{W}'$ and for the case that we are working in a $2^{|\mathcal{M}_1|}$-saturated enlargement.

With respect to this extended language, if you wish to except the possibility, a definition as to what constitutes a purely subtle alphabet symbol would need to be altered in the obvious fashion. Indeed, for T in the definition of P_m, we need to substitute $T' = i[\mathcal{W}']$. Then the altered definition would read that $r \in {}^*i[\mathcal{W}]$ is a *pure subtle alphabet symbol* if there exists an $m \in \mathbb{N}$ and $f \in {}^*(P_m)$, or if $m \in {}^*\mathbb{N} - \mathbb{N}$ an $f \in P_m$, and some $j \in {}^*\mathbb{R}$ such that $f(j) = r \notin i[\mathcal{W}']$. Notice that if one chooses, then r corresponds at an $r' \in {}^*\mathcal{W}'$. Further, some of the previous theorems also hold when the proofs are modified.

Although these extended languages are of interest to the mathematician, most of science is content with approximating a real number by means of a rational number. In all that follows, the cardinality of our language, if not denumerable, will be specified. All theorems from this book that are used to establish a result relative to a denumerable language will be stated without qualification. If a theorem has not been reestablished for a higher language but can be so reestablished, then the theorem will be termed an *extended* theorem.

9.3 General Paradigms.

There is the developmental paradigm, and for nondetailed descriptions the *general developmental paradigm.* But now we have something totally new — the *general paradigm.* It is important to note that the general paradigm is considered to be distinct from developmental paradigms, although certain results that hold for general paradigms will hold for developmental paradigms and conversely. For example, associated with each general paradigm $\mathrm{G_A}$ is an ultraword w_g such that the set $\mathbf{G_A} \subset {}^*\mathbf{S}(\{w_g\})$ and all other theorems relative to such ultrawords hold for general paradigms. The general paradigm is a collection of words that discuss, in general, the behavior of entities and other constituents of a natural system. They, usually, do not contain a time statement W_i as it appears in section 7.1 for developmental paradigm descriptions. Our interest in this section is relative to only two such general paradigms. The reader can easily generate many other general paradigms.

Let c' be a symbol that denotes some fixed real number and n' a symbol that denotes a natural number. [Note: what follows is easily extended to an extended language.] Suppose that you have a theory which includes each member of the following set (i suppressed.

$$\mathrm{G_A} = \{\mathrm{An|||elementary|||particle|||}\alpha(\mathrm{n}')\mathrm{|||with|||}$$

$$\text{kinetic|||energy|||c}'\text{+1/(n}').\ |\ n \in G \wedge n \neq 0\}, \tag{9.3.1}$$

where G is a denumerable subset of the real numbers. The set $\mathrm{G_A}$ is of particular interest when $G = \mathbb{N}$. Theories that include such sentences consider such particles as *free in space.*

Theorem 9.3.1 shows that it is necessary for propertons to exist.

Of particular interest is the composition of members of ${}^*\mathbf{G_A} - \mathbf{G_A}$. Notice that $|\mathbf{G_A}| = |G|$ since $z_1,\ z_2 \in \mathbf{G_A}$ and $z_1 \neq z_2$ iff $[x_1] = z_1,\ [x_2] = z_2,\ x_1(30) = x_1(2) \neq x_2(30) = x_2(2),\ x_1(2),\ x_2(2) \in G$. Now consider the bijection $K\colon \mathbf{G_A} \to G$.

Theorem 9.3.1 *The set* $[g] \in {}^*\mathbf{G_A} - \mathbf{G_A}$ *iff there exists a unique* $f \in {}^*(P_{55})$ *and* $\nu \in {}^*G - G$ *such that* $[g] = [f]$, *and* $f(55) = i(\mathrm{A}),\ f(54) = i(\mathrm{n}),\ f(53) = i(|||), \cdots, f(30) = f(2), \cdots, f(3) = i((), f(2) = \nu \in {}^*G - G \subset {}^*\mathbb{R} - \mathbb{R},\ f(1) = i()),\ f(0) = (.)$.

Proof. From the definition of $\mathrm{G_A}$ the sentence

$$\begin{gathered}\forall z(z \in \mathcal{E} \to ((z \in \mathbf{G_A}) \leftrightarrow \exists! x \exists! w((w \in G) \wedge (x \in P_{55}) \wedge (x \in z) \wedge \\ ((55, i(\mathrm{A})) \in x) \wedge ((54, i(\mathrm{n})) \in x) \wedge \cdots \wedge (x(30) = x(2)) \wedge \cdots \wedge \\ ((3, i(()) \in x) \wedge (x(2) = w) \wedge (K(z) = w) \wedge \\ ((1, i())) \in x) \wedge ((0, i(.)) \in x)))).\end{gathered} \tag{9.3.2}$$

holds in $\mathcal{M}$, hence in ${}^*\mathcal{M}$. From the fact that K is a bijection, it follows that ${}^*K[{}^*\mathbf{G_A} - \mathbf{G_A}] = {}^*G - G \subset {}^*\mathbb{R} - \mathbb{R}$. The result now follows from *-transfer. ▮

Using Theorem 9.3.1, each member of ${}^*\mathbf{G_A} - \mathbf{G_A}$, when interpreted by considering i^{-1}, has only two positions with a single missing object since positions 30 and 2 do not correspond to any symbol string in our language $\mathcal{W}$. This interpretation still retains a vast amount of content, however. For a specific member, you could substitute a new constructed symbol, not in $\mathcal{W}$, into these two missing positions. Depending upon what type of pure nonstandard number this inserted symbol represents, the content of such a sentence could be startling. Let Γ' be a nonempty set of new symbols disjoint from $\mathcal{W}$ and assume that Γ' is injectively mapped by H into ${}^*G - G$.

Although human ability may preclude the actual construction of more than denumerably many new symbols, you might consider this mapping to be onto if you accept the ideas of extended languages with a greater cardinality. As previously, denote these new symbols by γ'. Now let

$$\begin{gathered}\mathrm{G'_A} = \{\text{An|||elementary|||particle|||}\alpha(\gamma\prime)\text{|||with|||} \\ \text{kinetic|||energy|||c}'\text{+1/(}\gamma'\text{).}\ |\ H(\gamma') \in {}^*G - G\},\end{gathered} \tag{9.3.3}$$

This leads to the following interpretation stated in terms of describing sets.

(1) *The describing set* $\mathrm{G_A}$ *(mathematically) exists iff the describing set* $\mathrm{G'_A}$ *(mathematically) exists.*

9.4 Interpretations

Recall that the Natural world portion of the NSP-world model may contain *undetectable* objects, where “undetectable” means that there does not appear to exist human, or humanly constructible machine sensors that directly detect the objects or directly measure any of the objects physical properties. The rules of the scientific method utilized within the micro-world of subatomic physics allow all such undetectable Natural objects to be accepted as existing reality.[1] The properties of such objects are indirectly deduced from the observed properties of gross matter. In order to have indirect evidence of the objectively real existence of such objects, such indirectly obtained behavior will usually satisfy a specifically accepted model.

Although the numerical quantities associated with these undetectable Natural (i.e. standard) world objects, if they really do exist, cannot be directly measured, these quantities are still represented by standard mathematical entities. By the rules of correspondence for interpreting pure NSP-world entities, the members of G'_A must be considered as undetectable pure NSP-world objects, assuming any of them exist in this background world. On the other hand, when viewed within the EGS, any finite as well as many infinite subsets of G'_A are internal sets. Consequently, some finite collects of such objects *may be* assumed to indirectly effect behavior in the Natural world.

The concept of *realism* often dictates that all interpreted members of a mathematical model be considered as existing in reality. The philosophy of science that accepts only *partial realism* allows for the following technique. One can stop at any point within a mathematically generated physical interpretation. Then proceed from that point to deduce an intuitive physical theory, but only using other not interpreted mathematical formalism as auxiliary constructs or as catalysts. With respect to the NSP-world, another aspect of interpretation enters the picture. Assuming realism, then the question remains which, if any, of these NSP-entities actually indirectly influence Natural world processes? This interpretation process allows for the possibility that none of these pure NSP-world entities has any effect upon the standard world. These ideas should always be kept in mind.

If you accept that such particles as described by G_A can exist in reality, then the philosophy of realism leads to the next interpretation.

> (2) *If there exist elementary particles with Natural-system behavior described by* G_A*, then there exist pure NSP-world objects that display within the NSP-world behavior described by members of* G'_A.

The concept of absolute realism would require that the acceptance of the elementary particles described by G_A is indirect evidence for the existence of the G'_A described objects. I caution the reader that the interpretation we apply to such sets of sentences as G_A are only to be applied to such sets of sentences.

The EGS may, of course, be interpreted in infinitely many different ways. Indeed, the NSP-world model with its physical-type language can also be applied in infinitely many ways to infinitely many scenarios. I have applied it to such models as the GGU-model, among others. In this section, I consider another possible interpretation relative to those Big Bang cosmologies that postulate real objects at or near infinite temperature, energy or pressure. These theories incorporate the concept of the *initial singularity(ies).*

One of the great difficulties with many Big Bang cosmologies is that no meaningful physical interpretation for formation of the initial singularity is forthcoming from the theory itself. The fact that a proper and acceptable theory for creation of the universe requires that consideration not only be given to the moment of zero cosmic time but to what might have occurred “prior” to that moment in the nontime period is what partially influenced Wheeler to consider the concept of a *pregeometry.*[3], [4] It is totally unsatisfactory to dismiss such questions as “unmeaningful” simply because they cannot be discussed in your favorite theory. Scientists must search for a broader theory to include not only the question but a possible answer.

Although the initial singularity for a Big Bang type of state of affairs apparently cannot be discussed in a meaningful manner by many standard physical theories, it can be discussed by application of our NSP-world language. Let c′ be a symbol that represents any fixed real number. Define

$$\mathrm{G_B} = \{\mathrm{An|||elementary|||particle|||}\alpha(\mathrm{n}')\mathrm{|||with|||} \\ \mathrm{total|||energy|||c'{+}n'.} \mid n \in \mathbb{N}\}, \tag{9.4.1}$$

Application of Theorem 9.3.1 to $\mathrm{G_B}$ yields the set

$$\mathrm{G'_B} = \{\mathrm{An|||elementary|||particle|||}\alpha(\gamma')\mathrm{|||with|||} \\ \mathrm{total|||energy|||c'{+}}\gamma'. \mid \gamma \in {}^*\mathbb{N} - \mathbb{N}\}, \tag{9.4.2}$$

(3) *If there exist elementary particles with Natural-system behavior described by* $\mathrm{G_B}$, *then there exist pure NSP-world objects that display within the NSP-world behavior described by members of* $\mathrm{G'_B}$.

The particles being described by $\mathrm{G'_B}$ have various infinite energies. These infinite energies **do not** behave in the same manner as would the real number energy measures discussed in $\mathrm{G_B}$. As is usual when a metalanguage physical theory is generated from a formalism, we can further extend and investigate the properties of the $\mathrm{G'_B}$ objects by imposing upon them the corresponding behavior of the positive infinite hyperreal numbers. This produces some interesting propositions. Hence, we are able to use a nonstandard physical world language in order to give further insight into the state of affairs at or near a cosmic initial singularity. This gives *one* solution to a portion of the pregeometry problem. I point out that there are other NSP-world models for the beginnings of our universe, if there was such a beginning. Of course, the statements in $\mathrm{G'_B}$ need not be related at all to any Natural world physical scenario, but could refer only to the behavior of pure NSP-world objects.

Notice that Theorems such as 7.3.1 and 7.3.4 relative to the generation of developmental paradigms by ultrawords, also apply to general paradigms, where $\mathrm{M_d},\ \mathrm{M_B}, \mathrm{P_0}$ are defined appropriately. The following is a slight extension of Theorem 7.3.2 for general paradigms. Theorem 9.4.1 will also hold for developmental paradigms.

Theorem 9.4.1 shows that there exists an ultraword w such that the basic ultralogic ${}^*\mathbf{S}$ when applied to w yields not only a specific collection of elementary particles but also a collection of propertons. Further, it follows that there exists an ultraword w'' such that when ${}^*\mathbf{S}$ is applied to w'' the result is the collection all of the elementary particles that are claimed to exist and comprise the material portion of our universe.

Theorem 9.4.1 *Let* $\mathrm{G_C}$ *be any denumerable general paradigm. Then there exists an ultraword* $w \in {}^*\mathbf{P}_0$ *such that for each* $\mathbf{F} \in \mathbf{G_C},\ \mathbf{F} \in {}^*\mathbf{S}(\{w\})$ *and there exist infinitely many* $[g] \in {}^*\mathbf{G_C} - \mathbf{G_C}$ *such that* $[g] \in {}^*\mathbf{S}(\{w\})$.

Proof. In the proof of Theorem 7.3.2, it is shown that there exists some $\nu \in {}^*\mathbb{N} - \mathbb{N}$ such that ${}^*h[[0,\nu]] \subset {}^*\mathbf{S}(\{w\})$ and ${}^*h[[0,\nu]] \subset {}^*\mathbf{G_C}$. Since $|{}^*h[[0,\nu]]| \geq 2^{\aleph_0}$, then $|{}^*h[[0,\nu]] - h[\mathbb{N}]| \geq 2^{\aleph_0}$ for h is a bijection. This completes the proof. ∎

Corollary 9.4.1.1 *Theorem 9.4.1 holds, where* $\mathrm{G_C}$ *is replaced by a developmental paradigm.*

(4) *Let* $\mathrm{G_C}$ *be a denumerable general paradigm. There exists an intrinsic ultranatural process,* ${}^*\mathbf{S}$, *such that objects described by members of* $\mathrm{G_C}$ *are produced by* ${}^*\mathbf{S}$. *During this production, numerously many pure NSP-objects as described by statements in* ${}^*\mathbf{G_C} - \mathbf{G_C}$ *are produced.*

. . . .

CHAPTER 9 REFERENCES

1 Evans, R., *The Atomic Nucleus*, McGraw-Hill, New York, 1955.

2 Mendelson, E., *Introduction to Mathematical Logic*, Ed. 2, D. Van Nostrand Co., New York, 1979.

3 Misner, Thorne and Wheeler, *Gravitation*, W. H. Freeman, San Francisco, 1973.

4 Patton C. and A. Wheeler, Is physics legislated by cosmogony? in *Quantum Gravity*, ed. Isham, Penrose and Sciama, Oxford University Press, Oxford, 1977, 538–605.

10.1 More About Ultrawords.

Previously, we slightly investigated the composition of an ultraword $w \in {}^*\mathbf{M} - \mathbf{d}$. Using the idea of the minimum informal language $\mathrm{P_0} \subset \mathrm{P}$, where denumerable $\mathrm{B} = \mathrm{d}$ and P is a propositional language, our interest now lies in completely determining the composition of ${}^*\mathbf{S}(\{w\})$. [Note: since our language is informal axiom (3) and (4) are redundant in that superfluous parentheses have been removed.] First, two defined sets.

$$\mathrm{A} = \{\mathrm{x} \mid \mathrm{x} \in \mathrm{P_0} \text{ is an instance of an axiom for S}\} \tag{10.1.1}$$

$$\mathrm{C} = \{\mathrm{x} \mid \mathrm{x} \in \mathrm{P_0} \text{ is a finite } (\geq 1) \text{ conjunction of members of d}\} \tag{10.1.2}$$

Notice that it is also possible to refine the set C by considering C to be an ordered conjunction with respect to the ordering of the indexing set used to index members of d. Further, as usual, we have that A, C, d are mutually disjoint.

Theorem 10.1.1 shows that the contents of the ultranatural events that are produced when the ultralogic ${}^*\mathbf{S}$ is applied to an ultraword w cannot be described in any standard human language. However, certain general properties can be described.

Theorem 10.1.1 *Let* $w \in {}^*\mathbf{M_d} - {}^*\mathbf{d}$ *be an ultraword for infinite* $\mathbf{d} \subset {}^*\mathbf{S}(\{w\})$. *Then* ${}^*\mathbf{S}(\{w\}) = {}^*\mathbf{A} \cup Q_1 \cup d_1'$, *where for internal *- finite* d_1', $\mathbf{d} \subset d_1' \subset {}^*\mathbf{d}$ *and internal* $Q_1 \subset {}^*\mathbf{C}$ *is composed of *-finite* (≥ 1) *conjunctions (i.e.* $i(|||\mathrm{and}|||)$*) of distinct members of* d_1' *and* $w \in Q_1$. *Further, each member of* d_1' *and no other *-propositions is used to form the *-finite conjunctions in* Q_1, *the only *-propositions in* ${}^*\mathbf{S}(\{w\})$ *are those in* w, *and* ${}^*\mathbf{A}$, Q_1 *and* d_1' *are mutually disjoint.*

Proof. The intent is to show that if $\mathrm{w} \in \mathrm{M_d} - \mathrm{d}$, then $\mathrm{S}(\{\mathrm{w}\}) = \mathrm{A} \cup \mathrm{Q} \cup \mathrm{d}'$, where $\mathrm{Q} \subset \mathrm{C}$, finite $\mathrm{d}' \subset \mathrm{d}$ and Q is composed of finite (≥ 1) conjunctions of members of d', each member of d' is used to form these conjunctions and no other propositions.

Let J be the set of propositional atoms in the composite w. (**0**) Then $\mathrm{J} \subset \mathrm{S}(\{\mathrm{w}\})$. If K is the set of all propositional atoms in $\mathrm{S}(\{\mathrm{w}\})$, then $\mathrm{J} \subset \mathrm{K}$. Let $\mathrm{b} \in \mathrm{K} - \mathrm{J}$. It is obvious that $\mathrm{b} \notin \mathrm{S}(\{\mathrm{w}\})$ since otherwise $\{\mathrm{w}, \mathrm{b}\} \subset \mathrm{S_0}(\{\mathrm{w}\})$ but $\not\models_{\mathrm{S_0}} \mathrm{w} \to \mathrm{b}$. Thus, $\mathrm{J} = \mathrm{K}$. Consequently, $\mathrm{J} \subset \mathrm{S}(\{\mathrm{w}\})$, $\mathrm{J} \subset \mathrm{d}$ and there does not exists an $\mathrm{F} \in \mathrm{d} - \mathrm{J}$, such that $\mathrm{F} \in \mathrm{S}(\{\mathrm{w}\})$. (**1**) *Let* $\mathrm{J} = \mathrm{d}'$. *The only propositional atoms in*

$\mathrm{S}(\{\mathrm{w}\})$ *are those in w.* Obviously $\mathrm{A} \subset \mathrm{S}(\{\emptyset\})$.

Assume the language $\mathrm{P_0}$ is inductively defined from the set of atoms d. Recall that for our axioms $\mathcal{X} = \mathcal{D} \to \mathcal{F}$, the strongest connective in $\mathcal{X}$ is $\to$. While in $\mathcal{D}$, or $\mathcal{F}$ when applicable, the strongest connective is $\land$. Since $\emptyset \subset \{\mathrm{w}\}$, it follows that $\mathrm{S}(\{\mathrm{w}\}) = \mathrm{S}(\emptyset) \cup \mathrm{S}(\{\mathrm{w}\})$. Let $\mathrm{b} \in \mathrm{S}(\emptyset)$. The only steps in the formal proof for b contain axioms or follows from modus ponens. Suppose that step $\mathrm{B_k} = \mathrm{b}$ is the first modus ponens step obtained from steps $\mathrm{B_i}$, $\mathrm{B_j}$, $\mathrm{i}, \mathrm{j} < \mathrm{k}$, where

$\mathrm{B_i} = \mathrm{A} \to \mathrm{b},\ \mathrm{B_j} = \mathrm{A}$. The strongest connective for each axiom is $\to$. However, since $A \to b$ is an axiom, the strongest connective in A is $\wedge$. This contradicts the requirement that A must also be an axiom with strongest connect $\to$. Thus no modus ponens step can occur in a formal proof for b. Hence, (**2**) $\mathrm{A} = \mathrm{S}(\emptyset)$. (No modus ponens step can occur using two axioms.)

Let $\mathrm{B_k} = \mathrm{b_1} \in \mathrm{P_0}$ and suppose (a) that $\mathrm{b_1} = \mathrm{w}$, or (b) $\mathrm{b_1} \neq \mathrm{w}$ and is the first nonaxiom step that appears in a formal demonstration from the hypothesis w. Assume (b). Then all steps $\mathrm{B_i} \in \{\mathrm{w}\} \cup \mathrm{A},\ 0 \leq \mathrm{i} < \mathrm{k}$. Then the only way that $\mathrm{b_1}$ can be obtained is by means of modus ponens. However, all other steps, not including that which is w, are axioms. No modus ponens step can occur using two axioms. Thus one of the steps used for modus ponens must not be an axiom. The only nonaxiom that occurs prior to the step $\mathrm{B_k}$ is the step $\mathrm{B_m} = \mathrm{w}$. Hence, one of the steps required for $\mathrm{B_k}$ must be $\mathrm{B_m} = \mathrm{w}$. The other step must be an axiom of the form $\mathrm{w} \to \mathrm{b_1}$ and $\mathrm{b_1} \neq \mathrm{w}$. Thus, from the definition of the axioms (**3**) $\mathrm{b_1}$ is either a finite (≥ 1) conjunction of atoms in d', or a single member of d'. Assume strong induction. Hence, for $\mathrm{n} > 1$, statement (**3**) holds for all r, $1 \leq \mathrm{r} \leq \mathrm{n}$. A similar argument shows that (**3**) holds for the $\mathrm{b_{n+1}}$ nonaxiom step. Thus by induction, (**3**) holds for all nonaxiom steps.

Hence, there exists a $\mathrm{Q} \subset \mathrm{C}$ such that each member of Q is composed of finitely many (≥ 1) distinct members of d' and the set G(Q) of all the proposition atoms that appear in any member of $\mathrm{Q} = \mathrm{d}' = \mathrm{J}$ since $\mathrm{w} \in \mathrm{Q}$. Moreover, (**4**) $\mathrm{S}(\{\mathrm{w}\}) = \mathrm{A} \cup \mathrm{d}' \cup \mathrm{Q}$ and (**5**) A, d', Q are mutually disjoint.

$$\forall x(x \in \mathbf{M_d} - \mathbf{d} \to \exists y \exists z((y \in F(\mathbf{d})) \wedge (z \subset \mathbf{C}) \wedge (\mathbf{S}(\{x\}) =$$
$$\mathbf{A} \cup y \cup z) \wedge (\mathbf{A} \cap y = \emptyset) \wedge (\mathbf{A} \cap z = \emptyset) \wedge (x \in z) \wedge$$
$$(y \cap z = \emptyset) \wedge \mathbf{G}(z) = y)). \tag{10.1.3}$$

holds in $\mathcal{M}$, hence also in ${}^*\mathcal{M}$. So, let w be an ultraword. Then there exists internal $Q_1 \subset {}^*\mathbf{C},\ w \in Q_1$ and *- finite $d_1' \subset {}^*\mathbf{d}$ such that $\mathbf{d} \subset {}^*\mathbf{S}(\{w\}) = {}^*\mathbf{A} \cup d_1' \cup Q_1$; ${}^*\mathbf{A},\ d_1',\ Q_1$ are mutually disjoint and ${}^*G(Q_1) = d_1' = {}^*\mathbf{J}$. Hence, $\mathbf{d} \subset d_1'$.

Now to analyze the objects in Q_1. Let $\mathrm{d} = \{\mathrm{F_i} \mid \mathrm{i} \in \mathbb{N}\}$. Consider a bijection $h\colon \mathbb{N} \to \mathbf{d}$ defined by $h(n) = \mathbf{F_n} = [f]$, where $f \in T^0$ is the special member of $\mathbf{F_n}$ such that $f = \{(0, f(0))\},\ f(0) = i(\mathrm{F_n}) = q_n \in i[\mathrm{d}]$. From the above analysis, (A) $[g] \in \mathbf{S}(\{w\}) - \mathbf{A} - \mathbf{d},\ (w \in \mathbf{M_d} - \mathbf{d})$, iff there exist $k,\ j \in \mathbb{N}$ such that $k < j$ and $f_1' \in i[\mathrm{P_0}]^{2(j-k)}$ such that $[f_1'] = [g]$, and this leads to (B) that for each even $2p,\ 0 \leq 2p \leq 2(j-k)$; $f_1'(2p) = q_{k+p} \in i[\mathrm{P_0}] \subset i[\mathcal{W}],\ [(0, q_{k+p})] \in \mathbf{d}'$, all such q_{k+p} being distinct. For each odd $2p+1$ such that $0 \leq 2p+1 \leq 2(j-k),\ f_1'(2p+1) = i(|||\mathrm{and}|||)$. Also (C) $h(p) \in h[[k,j]]$ iff there exists an even $2p$ such that $0 \leq 2p \leq 2(j-k)$ and $f_1'(2p) = h(p) = q_{k+p} \in i[\mathrm{P_0}]$. [Note that 0 is considered to be an even number.]

By *-transfer of the above statements (A), (B) and (C), $[g] \in Q_1$ iff there exists some $j,\ k \in {}^*\mathbb{N},\ k < j$, and $f' \in {}^*(i[\mathrm{P_0}])^{2(j-k)}$ such that $[f'] = [g]$ and ${}^*h[[k,j]] \subset {}^*\mathbf{d}$. Moreover, each ${}^*h(r),\ r \in [k,j]$ is a distinct member of ${}^*\mathbf{d}$. The conjunction "codes" for $i(|||\mathrm{and}|||) \in i[\mathcal{W}]$ that are generated by each odd $2p+1$ are all the same and there are *-finitely many of them. Hence, Q_1 is the *-finite (≥ 1) conjunctions of distinct members of d_1', no other *-propositions are utilized and since ${}^*\mathbf{G}(Q_1) = d_1'$, all members of d_1' are employed for these conjunctions. This completes the proof. ▮

Corollary 10.1.1.1 *Let* $w \in {}^*\mathbf{M_d} - {}^*\mathbf{d}$ *be an ultraword for denumerable* d *such that* $\mathbf{d} \subset {}^*\mathbf{S}(\{w\})$. *Then* ${}^*\mathbf{S}(\{w\}) \cap \mathbf{P_0} = \mathbf{A} \cup \mathbf{Q} \cup \mathbf{d}$ *and* A, Q, d *are mutually disjoint. The set* Q *is composed of finite* ≥ 1 *conjunctions of members of* d *and all of the members of* d *are employed to obtain these conjunctions.*

Proof. Recall that due to the finitary character of our standard objects ${}^{\sigma}\mathbf{A} = \mathbf{A} = {}^{*}\mathbf{A} \cap \mathbf{P_0}$. In like manner, since $\mathbf{d} \subset d_1'$, $d_1' \cap \mathbf{P_0} = \mathbf{d}$. Now $\mathbf{P_0} \cap Q_1$ are all of the standard members of Q_1. For each $k \in {}^{*}\mathbb{N}$, ${}^{*}h(k) = \mathbf{F_k} \in {}^{*}\mathbf{d}$ and conversely. Further, $\mathbf{F_k} \in \mathbf{d}$ iff $k \in \mathbb{N}$. Restricting $k,\ j \in \mathbb{N}$ in the above theorem yields standard finite ≥ 1 conjunctions of standard members of d_1'; hence, members of $\mathbf{d}$. Since ultraword $w \in Q_1$, we know that there exists some $\eta \in {}^{*}\mathbb{N} - \mathbb{N}$ and $f_1' \in {}^{*}(i[\mathrm{P}_0])^{2\eta}$, where f_1' satisfies the *-transfer of the properties listed in the above theorem . Since finite conjunctions of standard members of d_1' are *-finite conjunctions of members of d_1' and $\mathbf{d} = \mathbf{d} \cap d_1'$, it follows that all possible finite conjunctions of members of $\mathbf{d}$ that are characterized by the function $f_1' \in i[\mathrm{P}_0]^{2(j-k)}$ are members of Q_1 for each such $j,\ k < \eta$. Also for such $j,\ k$ the values of f_1' are standard. On the other hand, any value of f_1' is nonstandard iff it corresponds to a member of $d_1' - \mathbf{d}$. Thus $Q_1 \cap \mathbf{P_0} = \mathbf{Q}$ and this completes the proof. ▮

If it is assumed that each member of d describes a Natural event (i.e. N-event) at times indicated by X_i, dropping the X_i may still yield a denumerable developmental paradigm without specifically generated symbols such as the "i." Noting that d_1' is *-finite and internal leads to the conclusion that we can have little or no knowledge about the word-like construction of each member of $d_1' - \mathbf{d}$. These pure nonstandard objects can be considered as describing pure NSP-world events, as will soon be demonstrated. Therefore, it is important to understand the following interpretation scheme, where descriptions are corresponded to events.

> *Standard or internal NSP-world events or sets of events are interpreted as directly or indirectly influencing N-world events. Certain external objects*, such as the standard part operator, among others, *are also interpreted as directly or indirectly influencing N-world events.*

Notice that standard events can directly or indirectly affect standard events. In the micro-world, the term *indirect evidence* or verification is a different idea than indirect influences. You can have direct or indirect evidence of direct or indirect influences when considered within the N-world. An indirect influence occurs when there exists, or there is assumed to exist, a mediating "something" between two events. Of course, indirect evidence refers to behavior that can be observed by normally accepted human sensors as such behavior is assumed to be caused by unobserved events. However, the evidence for pure NSP-world events that directly or indirectly influence N-world events must be indirect evidence under the above interpretation.

In order to formally consider NSP-world events for the formation of objective standard reality, proceed as follows: let $\mathcal{O}$ be the subset of $\mathcal{W}$ that describes those Natural events that are used to obtain developmental or general paradigms and the like. Let $\mathrm{E_j} \in \mathcal{O}$. Linguistically, assume that each $\mathrm{E_j}$ has the spacing symbol $|||$ immediately to the right. Thus within each T_i, there is a finite symbol string $\mathrm{F_i} = \mathrm{E_i} \in \mathcal{O}$ that can be joined by the justaposition (i.e. join) operation to other event descriptions. Assume that $\mathcal{W}_1$ is the set of nonempty symbol strings (with repetitions) formed (i.e. any finite permutation) from members of $\mathcal{O}$ by the join operation. These finite strings of symbols generate the basic elements for our partial sequences.

Obviously, $\mathcal{W}_1 \subset \mathcal{W}$. Consider $\mathrm{T}_i' = \{\mathrm{XW}_i \mid \mathrm{X} \in \mathcal{W}_1\}$ and note that in many applications the time indicator W_i need not be of significance for a given $\mathrm{E_j}$ in some of the strings. Obviously, $\mathrm{T}_i' \subset \mathrm{T}_i$ for each i. For our isomorphism i onto $i[\mathcal{W}]$, the following hold.

$$\forall y(y \in \mathcal{E} \to (y \in \mathbf{T}_i' \leftrightarrow \exists x \exists f \exists w((\emptyset \neq w \in F(i[\mathcal{O}])) \wedge (x \in \mathbb{N}) \wedge$$
$$(f(0) = i[W_i]) \wedge (f \in P) \wedge \forall z((z \in \mathbb{N}) \wedge (0 < z \leq x) \to$$

$$f(z) \in w) \wedge (f \in y)))). \tag{10.1.4}$$

$$\forall x(x \in \mathbb{N} \to \exists f \exists w((\emptyset \neq w \in F(i[\mathcal{O}])) \wedge (f \in P) \wedge$$
$$\forall z(z \in \mathbb{N} \to (0 < z \leq x \leftrightarrow f(z) \in w)))). \tag{10.1.5}$$

$$\forall w(\emptyset \neq w \in F(i[\mathcal{O}]) \to \exists x \exists f \exists y((f \in P) \wedge (x \in \mathbb{N}) \wedge (y \in \mathbf{T}'_i) \wedge$$
$$(f \in y) \wedge \forall z(z \in \mathbb{N} \to (0 < z \leq x \leftrightarrow f(z) \in w)))). \tag{10.1.6}$$

Since each finite segment of a developmental pardigm corresponds to a member of $\mathbf{T}'_i$, each nonfinite hyperfinite segment should correspond to a member of ${}^*(\mathbf{T}'_i) - \mathbf{T}'_i$ and it should be certain individual segments of such members of ${}^*(\mathbf{T}'_i) - \mathbf{T}'_i$ that correspond to the ultranatural events produced by an ultraword; UN-events that cannot be eliminated from an NSP-world developmental paradigms. [Note: for a scientific language, 10.1.4 - 10.1.6 and other such statements would correspond to an $\mathcal{W}'$ as generated by, at the least, a denumerable alphabet such as used in 9.2, 9.3.]

10.2 Laws and Rules.

One of the basic requirements of human mental activity is the ability to recognize the symbolic differences between finitely long strings of symbols as necessitated by our reading ability and to apply linguistic rules finitely many times. Gödel numberings specifically utilize such recognitions and the rules for the generation of recursive functions must be comprehended with respect to finitely many applications. Observe that Gödel number recognition is an "ordered" process while some fixed intuitive order is not necessary for the application of the rules that generate recursive functions.

In general, the simplest "rule" for ordered or unordered finite human choice, a rule that is assumed to be humanly comprehensible by finite recognition, is to simply *list* the results of our choice (assuming that they are symbolically representible in some fashion) as a partial finite sequence for ordered choice or as a finite set of finitely long symbol strings for an unordered choice. Hence, the end result for a finite choice can itself be considered as an algorithm "for that choice only." The next application of such a *finite choice rule* would yield the exact same partial sequence or choice set. Another more general rule would be a statement which would say that you should "choose a specific number of objects" from a fixed set (of statements). Yet, a more general rule would be that you simply are required to "choose a finite set of all such objects," where the term "finite" is intuitively known. Of course, there are numerous specifically described algorithms that will also yield finite choice sets.

From the symbol string viewpoint, there are trivial machine programmable algorithms that allow for the comparison of finitely long symbols with each member of a finite set of symbol strings B that will determine whether or not a specific symbol string is a member of B. These programs duplicate the results of human symbol recognition. As is well-known, there has not been an algorithm described that allows us to determine whether or not a given finite symbol string is a member of the set of all theorems of such theories as formal Peano Arithmetic. If one accepts Church's Thesis, then no such algorithm will ever be described.

Define the general finite human choice relation on a set A as $H_0(A) = \{(A, x) \mid x \in F_0(A)\}$, where F_0 is the finite power set operator (including the empty set = no choice is made). Obviously, the inverse H_0^{-1} is a function from $F(A)$ onto $\{A\}$. There are choice operators that produce sets with a specific number of elements that can be easily defined. Let $F_1(A)$ be the set of all singleton subsets of A. The axioms of set theory state that such a set of singleton sets exists. Define $H_1(A) = \{(A, x) \mid x \in F_1(A)\}$, etc. Considering such functions as defined on sets X that are members of a superstructure, then these relations are subsets of $\mathcal{P}(X) \times \mathcal{P}(X)$ and as such are also members of the superstructure.

The next discussion shows that the IUN-selection processes ${}^*\mathbf{C_i},\ i \geq 0$ exist as formal objects within the NSP-world.

Let A = P_0. Observe that ${}^\sigma\mathbf{H_0}(\mathbf{A}) = \{({}^*\mathbf{A}, x) \mid x \in F_0(\mathbf{A})\}$ and ${}^*\mathbf{H_i}(\mathbf{A}) = \{({}^*\mathbf{A}, x) \mid x \in {}^*(F_i(\mathbf{A}))\}$ $(i \geq 0)$. Now ${}^*(F_0(\mathbf{A})) = {}^*F_0({}^*\mathbf{A})$ is the set of all *-finite subsets of ${}^*\mathbf{A}$. On the other hand, for the $i > 0$ cardinal subsets, ${}^*(F_i(\mathbf{A})) = F_i({}^*\mathbf{A})$ for each $i \geq 1$. With respect to an ultraword w that generates the general and developmental paradigms, we know that $w \in {}^*\mathbf{P_0} - \mathbf{P_0}$ and that $({}^*\mathbf{P_0}, \{x\}) \in {}^*\mathbf{H_1}(\mathbf{P_0})$. The actual finite choice operators are characterized by th set-theoretic second projector operator P_2 as it is defined on $H_i(\mathrm{A})$. This operator embedded by the θ is the same as P_2 as it is defined on $\mathbf{H_i}(\mathbf{A})$. Thus, when $h = (\mathbf{A}, x) \in \mathbf{H_i}(\mathbf{A})$, then we can define $x = P_2(h) = C_i(h) = \mathbf{C_i}(h)$. The maps C_i and $\mathbf{C_i}$, formally defined below, are the specific finite choice operators. For consistency, we let C_i and $\mathbf{C_i}$ denote the appropriate finite choice operators for $H_i(\mathrm{A})$ and $\mathbf{H_i}(\mathbf{A})$, respectively.

Since the ${}^*P_2$ defined on say $\mathbf{H_i}(\mathbf{A})$ is the same as the set-theoretic second projection operator P_2, it would be possible to denote ${}^*\mathbf{C_i}$ as $\mathbf{C_i}$ on internal objects. For consistency, the notation ${}^*\mathbf{C_i}$ for these special finite choice operators is retained. Formally, let $\mathbf{C_i}\colon \mathbf{H_i}(\mathbf{A}) \to F_i(\mathbf{A})$. Observe that ${}^\sigma\mathbf{C_i} = \{{}^*(a,b) \mid (a,b) \in \mathbf{C_i}\} = \{(({}^*\mathbf{A}, b), b) \mid b \in F_i(\mathbf{A})\} \subset {}^*\mathbf{C_i}$; and, for $b \in F_i(\mathbf{A})$, $\mathbf{C_i}((\mathbf{A}, b)) = b$ implies that ${}^\sigma(\mathbf{C_i}((\mathbf{A}, b))) = \{{}^*a \mid a \in b\} = b$ from the construction of $\mathcal{E}$. Thus in contradistinction to the consequence operator, for each $({}^*\mathbf{A}, b) \in {}^\sigma\mathbf{H_i}$, the image $({}^\sigma\mathbf{C_i})(({}^*\mathbf{A}, b)) = {}^\sigma(\mathbf{C_i}((\mathbf{A}, b))) = ({}^*\mathbf{C_i})(({}^*\mathbf{A}, b)) = b$. Consequently, the set map ${}^\sigma\mathbf{C_i}\colon {}^\sigma\mathbf{H_i} \to F_i(\mathbf{A}) = {}^\sigma(F_i(\mathbf{A}))$ and ${}^*\mathbf{C_i} \mid {}^\sigma\mathbf{H_i} = {}^\sigma\mathbf{C_i}$. Finally, it is not difficult to extend these finite choice results to general internal sets.

In the proofs of such theorems as 7.2.1, finite and other choice sets are selected due to their set-theoretic existence. The finite choice operators C_i are not specifically applied since these operators are only intended as a mathematical model for apparently effective human processes — procedures that generate acceptable algorithms. As is well-known, there are other describable rules that also lead to finite or infinite collections of statements. Of course, with respect to a Gödel encoding i for the set of all words $\mathcal{W}$ the finite choice of readable sentences in $\mathcal{E}$ is one-to-one and effectively related to a finite and, hence, recursive subset of $\mathbb{N}$.

From this discussion, the descriptions of the finite choice operators would determine a subset of the set of all algorithms ("rules" written in the language $\mathcal{W}$) that allow for the selection of readable sentences. Notice that before algorithms are applied there may be yet another set of readable sentences that yields conditions that must exist prior to an application of such an algorithm and that these application rules can be modeled by members of $\mathcal{E}$.

In order to be as unbiased as possible, it has been required for N-world applications that the set of all frozen segments be infinite. Thus, within the proof of Theorem 7.2.1, every N-world developmental, as well as a general paradigm, is a proper subset of a *-finite NSP-world paradigm, and the *-finite paradigm is obtained by application of the *-finite choice operator ${}^*\mathbf{C_0}$. As has been shown, such *-finite paradigms contain pure unreadable (subtle) sentences that may be interpreted for developmental paradigms as pure refined NSP-world behavior and for general paradigms as specific pure NSP-world ultranatural events or objects.

Letting Γ correspond to the formal theory of Peano Arithmetic, then assuming Church's Thesis, there would not exist a N-world algorithm (in any human language) that allows for the determination of whether or not a statement F in the formal language used to express Γ is a member of Γ. By application of the *-finite choice operator $^*\mathbf{C_0}$, however, there does exist a *-finite Γ' such that $^\sigma\Gamma = \Gamma \subset \Gamma'$ and, hence, within the NSP-world a "rule" that allows the determination of whether or not $\mathrm{F} \in \Gamma'$. If such internal processes mirror the only allowable procedures in the NSP-world for such a "rule," then it might be argued that we do not have an effective NSP-world process that determines whether or not F is a member of Γ for Γ is external.

As previously alluded to at the beginning of this section, when a Gödel encoding i is utilized with the N-world, the injection i is not a surjection. When such Gödel encodings are studied, it is usually *assumed*, without any further discussion, that there is some human mental process that allows us to recognize that one natural number representation (whether in prime factored form or not) is or is not distinct from another such representation. It is not an unreasonable assumption to assume that the same *effective* (but external) process exists within the NSP-world. Thus within the NSP-world there is a "process" that determines whether or not an object is a member of $^*\mathbb{N} - \mathbb{N} = \mathbb{N}_\infty$ or $\mathbb{N}$. Indeed, from the ultraproduct construction of our nonstandard model, a few differences can be detected by the human mathematician. Consequently, this assumed NSP-world effective process would allow a determination of whether or not $\mathbf{F} = [f_m]$ is a member of Γ by recalling that $f_m \in P_m$ signifies that $[f_m] \in {}^*\Gamma - \Gamma$ implies $m \in \mathbb{N}_\infty \simeq {}^*(i[\mathcal{W}]) - i[\mathcal{W}]$.

The above NSP-world recognition process is equivalent, as defined in Theorem 7.2.1, to various applications of a single (external) set-theoretic intersection. Therefore, there are internal processes, such as $^*\mathbf{C_0}$, that yield pure NSP-world developmental paradigms and a second (external) but acceptable NSP-world effective process that produces specific N-world objects. Relative to our modeling procedures, it can be concluded that both of these processes are intrinsic ultranatural processes.

With respect to Theorem 10.1.1, the NSP-world developmental or general paradigm generated by an ultraword is *-finite and, hence, specifically NSP- world obtainable prior to application of $^*\mathbf{S}$ through application of $^*\mathbf{C_0}$ to $^*\mathbf{d}$. However, this composition can be reversed. The NSP-world (IUN) process $^*\mathbf{C_1}$ can be applied to the appropriate $^*\mathbf{M_d}$ type set and an appropriate ultraword $w \in {}^*\mathbf{M_d}$ obtained. Composing $^*\mathbf{C_1}$ with $^*\mathbf{S}$ would yield d'_1 in a slightly less conspicuous manner. Obviously, different ultrawords generate different standard and nonstandard developmental or general paradigms.

To complete the actual mental-type processes that lead to the proper ordered event sequences, the above discussion for the finite choice operators is extended to the human mental ability of ordering a finite set in terms of rational number subscripts. New choice operators are defined that model not just the selection of a specific set of elements that is of a fixed finite cardinality but also choosing the elements in the required rational number ordering. The ultrawords w that exist are *-finite in length. By application of the inverses of the f and τ functions of section 7.1, where they may be considered as extended standard functions $^*\!f$ and $^*\tau$, there would be from analysis of extended theorem 7.3.2 a hyperfinite set composed of standard or nonstandard frozen segments contained in an ultraword. Further, in theorem 7.3.2, the chosen function f does not specifically differentiate each standard or nonstandard frozen segment with respect to its "time" stamp subscript. There does exist, however, another function in the *-equivalence class $[g] = w$ that will make this differentiation. It should not be difficult to establish that after application of the ultralogic $^*\mathbf{S}$, there is applied an appropriate mental-like hyperfinite ordered choice operator (an IUN-selection process) and that this would yield that various types of event sequences. Please note that each event sequence has a beginning point of observation. This point of observation need not indicate the actual moment when a specific Natural-system began its development.

In the following discussion, we analyze how ultranatural laws aid in the production of Natural and UN-event sequences, and develop an external unification of all physical theories.

Various subdevelopmental (or subgeneral) paradigms $\mathrm{d_i}$ are obtained by considering the actual descriptive content (i.e. events) of specific theories Γ_i that are deduced from hypotheses η_i, usually, by finitary consequence operators $\mathrm{S_i}$ (the inner logics) that are compatible with S. In this case, $\mathrm{d_i} \subset \mathrm{S_i}(\eta_\mathrm{i})$. It is also possible to include within $\{\mathrm{d_i}\}$ and $\{\eta_\mathrm{i}\}$ the assumed descriptive chaotic behavior that seems to have no apparent set of hypotheses except for that particular developmental paradigm itself and no apparent deductive process except for the identity consequence operator. In this way, such scientific nontheories can still be considered as a formal theory produced by a finitary consequence operator applied to an hypothesis. Many of these hypotheses η_i contain the so-called natural laws (or first-principles) peculiar to the formal theories Γ_i and the theories language, where it is assume that such languages are at least closed under the informal conjunction and conditional.

Consider each η_i to be a general paradigm. For the appropriate M type set constructed from the denumerable set $\mathrm{B} = \{\bigcup\{\mathrm{d_i}\}\cup(\bigcup\{\eta_\mathrm{i}\})$, redefine $\mathrm{M_B}$ to be the smallest subset of $\mathrm{P_0}$ containing B and closed under finite (≥ 0) conjunction. (The usual type of inductively defined $\mathrm{M_B}$.) Then there exist ultrawords $w_i \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ such that $\eta_\mathbf{i} \subset {}^*\mathbf{S}(\{w_i\})$ (where due to parameters usually *ultranatural laws* exist in ${}^*\mathbf{S}(\{w_i\}) - \eta_\mathbf{i}$) and $\mathbf{d_i} \subset {}^*\mathbf{S}(\{w_i\})$. Using methods such as those in Theorem 7.3.4, it follows that there exists some $w" \in {}^*\mathbf{M_B} - \mathbf{M_B}$ such that $w_i \in {}^*\mathbf{S}(\{w"\})$ and, consequently, $\eta_\mathbf{i} \cup \mathbf{d_i} \subset {}^*\mathbf{S}(\{w"\})$. Linguistically, it is hard to describe the ultraword $w"$. Such a $w"$ might be called an *ultimate ultranatural hypothesis* or *the ultimate building plain.*

Remark. It is not required that the so-called Natural laws that appear in some of the η_i be either cosmic time or universally applicable. They could refer only to local first-principles. It is not assumed that those first-principles that display themselves in our local environment are universally space-time valid.

Since the consequence operator S is compatible with each $\mathrm{S_i}$, it is useful to proceed in the following manner. First, apply the IUN-process ${}^*\mathbf{S}$ to $\{w"\}$. Then $\mathbf{d_i} \cup \eta_\mathbf{i} \subset {}^*\mathbf{S}(\{w"\})$. It now follows that $\mathbf{d_i} \cup \eta_\mathbf{i} \subset {}^*\mathbf{S}(\{w_i\}) \subset {}^*\mathbf{S_i}({}^*\mathbf{S}(\{w_i\})) \subset {}^*\mathbf{S_i}({}^*\mathbf{S_i}(\{w_i\})) = {}^*\mathbf{S_i}(\{w_i\})$. Observe that for each $\mathrm{a} \in \Gamma_\mathrm{i}$ there exists some finite $\mathrm{F_i} \subset \eta_\mathrm{i}$ such that $\mathbf{a} \in \mathbf{S_i}(\mathbf{F_i})$. However, $\mathbf{F_i} \subset \eta_\mathbf{i}$ for each member of $F(\eta_\mathrm{i})$ implies that $\mathbf{a} \in {}^*\mathbf{S_i}(\mathbf{F_i}) \subset {}^*\mathbf{S_i}({}^*\mathbf{S}(\{w_i\}))$. Consequently, $\mathbf{\Gamma_i} \subset {}^*\mathbf{S_i}({}^*\mathbf{S}(\{w_i\}))$. The ultimate ultraword suffices for the descriptive content and inner logics associated with each theory Γ_i.

We now make the following observations relative to "rules" and deductive logic. It has been said that science is a combination of empirical data, induction and deduction, and that you can have the first two without the last. That this belief is totally false should be self-evident since the philosophy of science requires its own general rules for observation, induction, data collection, proper experimentation and the like. All of these general rules require logical deduction for their application to specific cases — the metalogic. Further, there are specific rules for linguistics that also must be properly applied prior to scientific communication. Indeed, we cannot even open the laboratory door — or at least describe the process — without application of deductive logic. The concept of deductive logic as being the patterns our "minds" follow and its use exterior to the inner logic of some theory should not be dismissed for even the (assumed?) mental methods of human choice that occur prior to communicating various scientific statements and descriptions.

Finally, with respect to the hypothesis rule in [9], it might be argued that we can easily analyze the specific composition of all significant ultrawords, as has been previously done, and the composition of the nonstandard extension of the general paradigm. Using this assumed analysis and an additional alphabet, one *might* obtain specific information about pure NSP-world ultranatural laws or refined behavior. Such an argument would seem to invalidate the cautious hypothesis rule and lead to appropriate speculation. However, such an argument would itself be invalid.

Let $\mathcal{W}_1$ be an infinite set of meaningful readable sentences for some description and assume that $\mathcal{W}_1$ does not contain any infinite subset of readable sentences each one of which contains a mathematically interpreted entry such as a real number or the like. Since $\mathcal{W}_1 \subset \mathcal{W}$ and the totality $\mathrm{T}_i = \{\mathrm{XW}_i \mid \mathrm{X} \in \mathcal{W}\}$ is denumerable, the subtotality $\mathrm{T}'_i = \{\mathrm{XW}_i \mid \mathrm{X} \in \mathcal{W}_1\}$ is also denumerable. Hence, the external cardinality of ${}^*\mathbf{T}'_\mathbf{i} \geq 2^{|\mathcal{M}|}$.

Consider the following sentence

$$\forall z(z \in i[\mathcal{W}_1] \to \exists y \exists x((y \in i[\mathcal{W}]^{[0,1]}) \land (x \in \mathbf{T}'_\mathbf{i}) \land (y \in x) \land ((0, i(\mathrm{W}_i)) \in y) \land ((1, z) \in y)))). \tag{10.2.1}$$

By *-transfer and letting "z" be an element in ${}^*(i[\mathcal{W}_1]) - i[\mathcal{W}_1]$ it follows that we can have little knowledge about the remaining and what must be unreadable portions that take the "X" position. If one assumes that members of $\mathcal{W}_1$ are possible descriptions for possible NSP-world behavior at the time t_i, then it may be assumed that at the time t_i the members of ${}^*\mathbf{T}'_\mathbf{i} - \mathbf{T_i}$ describe NSP-world behavior at NSP- world (and N-world) time t_i. Now as i varies over ${}^*\mathbb{N}$, pure nonstandard subdevelopmental paradigms (with or without the time index statement W_i) exist with members in ${}^*\mathcal{T}$ and may be considered as descriptions for time refined NSP-world behavior, especially for a NSP-world time index $i \in \mathbb{N}_\infty$.

CHAPTER 10 REFERENCES

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17 Note that the NSP-world model is not a local hidden variable theory.

. . . .

In the next section, it is shown how the mathematical structure predicts the possible existence of the ultra-properton and how, by the very simple process of ultrafinite combinations, all of the Natural universe assumed fundamental entities can be produced. Also, this allows for the construction of infinitely many distinct collections of fundamental entities for distinct universes. Further, it allows the virtual particle concept to be replaced by ultrafinite combinations.

11.1 Propertons (Subparticles)).

What is a properton? Or, what is an infant? Or, better still, what is a thing? I first used the name infant for these strange objects. I then coined the term properton. It would have been better to have simply called them "things." As stated in [9], these objects are not to be described in terms of any geometric configuration. These multifaceted things, these propertons, are not to be construed as either particles nor waves nor quanta nor anything that can be represented by some fixed imagery. propertons are to be viewed only operationally. Propertons are only to be considered as represented by a *-finite sequence $\{a_i\}_{i=1}^n,\ n \in {}^*\mathbb{N}$, of hyperreal numbers. Indeed, the idea of the n-tuple $(a_1, a_2, \ldots, a_i, \ldots)$ notation is useful and we assume that n is a fixed member of $\mathbb{N}_\infty$. The language of coordinates for this notation is used, where the i'th coordinate means the i'th value of the sequence. Obviously, 0 is not a domain member for our sequential representation.

The first coordinate a_1 is a "naming" coordinate. The remaining coordinates are used to represent various real numbers, complex numbers, vectors, and the like physical qualities needed for different physical theories. For example, $a_2 = 1$ might be a counting coordinate. Then $a_i,\ 3 \leq i \leq 6$ are hyperreal numbers that represent NSP-world coordinate locations of the properton named by a_1 — $a_7,\ a_8$ represent the positive or negative charges that can be assigned to every properton — $a_9,\ a_{11},\ a_{13}$ hyperreal representations for the inertial, gravitational and intrinsic (rest) mass. For vector quantities, continue this coordinate assignment and assign specific coordinate locations for the vector components. So as not to be biased, include as other coordinates hyperreal measures for qualities such as energy, apparent momentum, and all other physical qualities required within theories that must be combined in order to produce a reasonable description for N-world behavior. For the same reason, we do not assume that such N-world properties as the uncertainty principle hold for the NSP-world. (See note (2) on page 60.)

It is purposely assumed that the qualities represented by the coordinate $a_i,\ i \geq 3$ are not inner-related, in their basic construction, by any mathematical relation since it is such inner-relations that are assumed to mirror the N-world laws that govern the development of not only our present universe but previous as well as future developmental alterations. The same remarks apply to any possible and distinctly different universes that may or not occur. Thus, for these reasons, we view the properton as being totally characterized by such a sequence $\{a_i\}$ and always proceed cautiously when any attempt is made to describe all but the most general properton behavior. Why have we chosen to presuppose that propertons are characterized by sequences, where the coordinates are hyperreal numbers?

For chapters 11, 12 assume EGS. Let r be a positive real number. The number r can be represented by a decimal number, where for uniqueness, the repeated 9s case is used for all terminating decimals. From this, it is seen that there is a sequence S_i of natural numbers such that $S_i/10^i \to r$. Consequently, for any $\omega \in \mathbb{N}_\infty = {}^*\mathbb{N} - \mathbb{N}$, it follows that $\pm\ {}^*S_\omega/10^\omega \in \mu(\pm r)$, where ${}^*S_\omega \in {}^*\mathbb{N}$ and $\mu(\pm r)$ is the monad about r. In [9], it is assumed that each coordinate $a_i,\ i \geq 3$ is characterized by the numerical quantity $\pm 10^{-\omega},\ \omega \in \mathbb{N}_\infty$. Obviously, we need not confine ourselves to the number $10^{-\omega}$. Indeed, the next theorem has interesting applications relative to such a selection.

Theorem 11.1.1 *Let $\omega \in \mathbb{N}_\infty$. Then for each $r \in \mathbb{R}$ there exists an $x \in \{m/\omega \mid (m \in {}^*\mathbf{Z}) \wedge (|m| < \omega^2)\}$ such that $x \approx r$ (i.e. $x \in \mu(r)$.)*

Proof. Let $r \in \mathbb{R}$. Then there exists some integer $n \in \mathbf{Z}$ such that $n \leq r < n+1$. Partition $[n, n+1]$ as follows: let $0 \neq m \in \mathbb{N}$ and consider $n \leq n + 1/m \leq \cdots \leq n + (m-1)/m \leq n+1$. Then there exists a unique $a \in \{0, 1, \ldots, m-1\}$ such that $r \in [n+a/m, n+(a+1)/m)$. For $m = 1$, let $S_1 = n = f_1/1$. For $m \geq 2$, if $a \in \{0, 1, \ldots, m-2\}$, then let $S_m = n+(a+1)/m = (nm+a+1)/m = f_m/m$; if $a = m-1$, then let $S_m = n + (m-1)/m = (nm+m-1)/m = f_m/m$. This yields two sequences $S\colon \mathbb{N} \to \mathbf{Q}$ and $f\colon \mathbb{N} \to \mathbf{Z}$. It follows easily that $S_i \to r$. Hence, for each $\omega \in \mathbb{N}_\infty$, ${}^*S_\omega = {}^*f_\omega/\omega \approx r$ and ${}^*f_\omega \in {}^*\mathbf{Z}$. Observe that ${}^*f_\omega/\omega$ is a finite (i.e. limited) number. Hence, $|{}^*f_\omega/\omega| < \omega$ entails that $|{}^*f_\omega| < \omega^2$. Therefore, ${}^*f_\omega/\omega \in \{m/\omega \mid (m \in {}^*\mathbf{Z}) \wedge (|m| < \omega^2)\}$. ∎

Corollary 11.1.1.1 *Let $\omega \in \mathbb{N}_\infty$. Then there exists a sequence $f\colon \mathbb{N} \to \mathbf{Z}$ such that ${}^*f_\omega/{}^*m_\omega \in \mu(r)$ for each $r \in \mathbb{R}$. Also, there exists a sequence $g\colon \mathbb{N} \to \mathbb{N}$ such that for each real $r \geq 0$, $(\pm\ {}^*g_\omega/\omega) \approx \pm r$.*

For the *ultra-properton*, each coordinate $a_i = 1/10^\omega\ i \geq 3$ and odd, $a_i = -1/10^\omega\ i \geq 4$ and even, $\omega \in \mathbb{N}_\infty$. From the above theorem, the choice of $10^{-\omega}$ as the basic numerical quantity is for convenience only and is not unique accept in its infinitesimal character. Of course, the sequences chosen to represent the ultra-properton are pure internal objects and as such are considered to directly or indirectly affect the N-world. Why might the *-finite "length" of such propertons (here is where we have replaced the NSP-world entity by its corresponding sequence) be of significance?

First, since our N-world languages are formed from a finite set of alphabets, it is not unreasonable to assume that NSP-world "languages" are composed from a *-finite set of alphabets. Indeed, since it should not be presupposed that there is an upper limit to the N-world alphabets, it would follow that the basic NSP-world set of alphabets is an infinite *-finite set. Although the interpretation method that has been chosen does not require such a restriction to be placed upon NSP-world alphabets, it is useful, for consistency, to assume that descriptions for substratum processes that affect, in either a directly or indirectly detectable manner, N-world events be so restricted. For the external NSP-world viewpoint, all such infinite *-finite objects have a very significant common property. [Note relative to the basic book "The Theory of Ultralogics." In what follows $\mathcal{M}_1$ is the standard superstructure constructed on page 76 and not the object defined on page 57.]

Theorem 11.1.2 *All infinite *-finite members of our (ultralimit) model ${}^*\mathcal{M}_1$ have the same external cardinality which is $\geq |\mathcal{M}_1|$.*

Proof. Hanson [8] and Zakon [16] have done all of the difficult work for this result to hold. First, one of the results shown by Henson is that all infinite *-finite members of our ultralimit model have the same external cardinality. Since our model is a comprehensive enlargement, Zakon's theorem

3.8 in [16] applies. Zakon shows that there exists a *-finite set, A, such that $|A| \geq |\mathcal{M}_1| = |\mathcal{R}|$. Since A is infinite, Hanson's result now implies that all infinite *-finite members of our model satisfy this inequality.∎

For an extended infinite standard set ${}^*A$ it is well-known that $|{}^*A| \geq 2^{|\mathcal{M}_1|}$. One may use these various results and establish easily that there exist more than enough propertons to obtain all of the cardinality statements relative to the three substratum levels that appear in [9] even if we assume that there are a continuum of finitely many properton qualities that are needed to create all of the N-world.

Consider the following infinite set of statements expressed in an extended alphabet.

$$\begin{aligned} \mathrm{G_A} = \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||} \\ &\mathrm{total|||energy|||c'{+}1/(n').} \mid ((i,j,n) \in \\ &\mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+) \wedge (1 \leq k \leq m)\}, \end{aligned} \tag{11.1.1}$$

where $\mathbb{N}^+$ is the set of all nonzero natural numbers and $m \in \mathbb{N}^+$. Applying the same procedure that appears in the proof of Theorem 9.3.1 and with a NSP-world alphabet $\mathcal{W}'$, we obtain

$$\begin{aligned} \mathrm{G'_A} = \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||} \\ &\mathrm{total|||energy|||c'{+}1/(n').} \mid ((i,j,n) \in \\ &{}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+) \wedge (1 \leq k \leq m)\}, \end{aligned} \qquad (11.1.2$$

Assume that there is at least one type of elementary particle with the properties stated in the set $\mathrm{G_A}$. It will be shown in the next section that within the NSP-world there may be simple properties that lead to N-world energy being a manifestation of mass. For $c = 0$, we have another internal set of descriptions that forms a subset of $\mathrm{G'_A}$.

$$\begin{aligned} \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||} \\ &\mathrm{total|||energy|||c'{+}1/(10^{\gamma'}).} \mid ((i,j,\gamma) \in \\ &{}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+) \wedge (1 \leq k \leq m)\}, \end{aligned} \tag{11.1.3}$$

For our purposes, (11.1.3) leads immediately to the not ad hoc concept of propertons with infinitesimal proper mass. As will be shown, such infinitesimal proper mass can be assumed to characterize any possible zero proper mass N-world entity. The set $\mathrm{G'_A}$ has meaning if there exists at least one natural entity that can possess the energy expressed by $\mathrm{G_A}$, where this energy is measured in some private unit of measure.

Human beings combine together finitely many sentences to produce comprehensible descriptions. Moreover, all N-world human construction requires the composition of objectively real N-world objects. We model the idea of *finite composition* or *finite combination* by an N-world process. This produces a corresponding NSP-world intrinsic ultranatural process *ultrafinite composition* or *ultrafinite combination* that can either directly or indirectly affect the N-world, where its effect is indirectly inferred.

Let the index j vary over a hyperfinite interval and fix the other indices. Then the set of sentences

$$\begin{aligned} \mathrm{G''_A} = \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||} \\ &\mathrm{total|||energy|||c'{+}1/(n').} \mid (j \in {}^*\mathbb{N}^+) \\ &\wedge(1 \leq j \leq \lambda)\}, \end{aligned} \tag{11.1.4}$$

where $\lambda \in {}^*\mathbb{N}^+$, $3 \leq i \in {}^*\mathbb{N}$, $n \in \mathbb{N}_\infty$ and $1 \leq k \leq m$, forms an internal linguistic object that can be assumed to describe a hyperfinite collection of ultranatural entities. Each member of $\mathrm{G}''_{\mathrm{A}}$ has the i'th coordinate that measures the proper mass and is infinitesimal (with respect to NSP-world private units of measure). In the N-world, finite combinations yield an event. Thus, with respect to such sets as $\mathrm{G}''_{\mathrm{A}}$, one can say that there are such N-world events iff there are ultrafinite combinations of NSP-world entities. And such ultrafinite combinations yield a NSP-world event that is an ultranatural entity.

Associated with such ultrafinite combinations for the entities described in $\mathrm{G}''_{\mathrm{A}}$ there is a very significant procedure that yields the i'th coordinate value for the entity obtained by such ultrafinite combinations. Such entities are called *intermediate propertons.* Let $m_0 \geq 0$ be the N-world proper mass for an assumed elementary particle denoted by k$'$. If $m_0 = 0$, then let $\lambda = 1$. Otherwise, from Theorem 11.1.1, we know that there is a $\lambda \in {}^*\mathbb{N}$ such that $\lambda/(10^\omega) \in \mu(m_0)$, where $\omega \in \mathbb{N}_\infty$ and since $m_0 \neq 0$, $\lambda \in \mathbb{N}_\infty$. Consequently, for $b_n = 10^{-\omega}$, the *-finite sum

$$\sum_{n=1}^{\lambda} b_n = \sum_{n=1}^{\lambda} \frac{1}{10^\omega} = \frac{\lambda}{10^\omega} \tag{11.1.5}$$

has the property that $\mathtt{st}(\sum_{n=1}^{\lambda} 1/(10^\omega)) = m_0$. (Note the special summation notation for a constant summand.) The standard part operator st is an important external operator that is a continuous [11] NSP-world process that yields N-world effects. The appropriate interpretation is that

> *ultrafinite combinations of ultra-propertons yield an intermediate properton that, after application of the standard part operator, has the same effect as an elementary particle with proper mass m_0.*

An additional relevant idea deals with the interpretation that the *-finite set $\mathrm{G}''_{\mathrm{A}}$ exists at, say, nonstandard time, and that such a set is manifested at standard time when the operator st is applied. The standard part operator is one of those external operators that can be indirectly detected by the presence of elementary particles with proper mass m_0.

The above discussion of the creation of intermediate propertons yields a possible manner in which ultra-propertons are combined within the NSP-world to yield appropriate energy or mass coordinates for the multifaceted propertons. But is there an indication that all standard world physical qualities that are denoted by qualitative measures begin as infinitesimals?

Consider the infinitesimal methods used to obtain such things as the charge on a sphere, charge density and the like. In all such cases, it is assumed that charge can be infinitesimalized. In 1972, it was shown how a classical theory for the electron, when infinitesimalized, leads to the point charge concept of quantum field theory and then how the *-finite many body problem produced the quasi-particle. [15] Although this method is not the same as the more general and less ad hoc properton approach, it does present a procedure that leads to an infinitesimal charge density and then, in a very ad hoc manner, it is assumed that there are objects that when *-finitely combined together entail a real charge and charge density. Further, it is the highly successful use of the modeling methods of infinitesimal calculus over hundreds of years that has lead to our additional presumption that all coordinates of the basic sequential properton representation are a $\pm$ fixed infinitesimal.

In order to retain the general independence of the coordinate representation, *independent *-finite coordinate summation* is allowed, recalling that such objects are to be utilized to construct many possible universes. [This is the same idea as *-finitely repeated simple affine or linear transformations.] Thus, distinct from coordinatewise addition, *-finitely many such sequences can be added together by means of a fixed coordinate operation in the following sense. Let $\{a_i\}$ represent an ultra-properton. Fix the coordinate j, then the sequence $\{c_i\},\ c_i = a_i,\ i \neq j$ and $c_j = 2a_j$ forms an intermediate properton. As will be shown, it is only after the formation of such intermediate propertons that the customary coordinatewise addition is allowed and this yields, after the standard part operator is applied, representations for elementary particles. Hence, from our previous example, we have that ultrafinite combinations of ultra-propertons yield propertons with "proper mass" $\lambda/(10^\omega) \approx m_0$ while all other coordinates remain as $\pm 10^\omega$. This physical-like process is not a speculative ad hoc construct, but, rather, it is modeled after what occurs in our observable natural world. Intuitively, this type of summation is modeled after the process of inserting finitely many pieces of information (mail) into a single "postal box," where these boxes are found in rectangular arrays in post offices throughout the world.

Now other ultra-propertons are ultrafinitely combined and yield for a specific coordinate the $\pm$ unite charge or, if quarks exist, other N-world charges, while all other coordinates remain fixed as $\pm 1/(10^\omega)$, etc. Rationally, how can one conceive of a combination of these intermediate propertons, a combination that will produce entities that can be characterized in a standard particle or wave language?

Recall that a finite summation is a *-finite summation within the NSP-world. Therefore, a finite combination of intermediate propertons is an allowed internal process. [Note that external processes are always allowed but with respect to our interpretation procedures we always have direct or indirect knowledge relative to application of internal processes. Only for very special and reasonable external processes do we have direct or indirect knowledge that they have been applied.] Let $\gamma_i \in \mu(0),\ i = 1, \ldots, n$. Then $\gamma_1 + \cdots \gamma_n \in \mu(0)$. The final stage in properton formation for our universe — the final stage in particle or wave substratum formation — would be *finite coordinatewise* summation of finitely many intermediate propertons. This presupposes that the N-world environment is characterized by but finitely many qualities that can be numerically characterized. This produces the following type of coordinate representation for a specific coordinate j after n summations with n other intermediate propertons that have only infinitesimals in the j coordinate position.

$$\sum_{i=1}^{\lambda}(1/(10^\omega) + \sum_{i=1}^{n}\gamma_i. \tag{11.1.6}$$

Assuming λ is one of those members of $\mathbb{N}_\infty$ or equal to 1 as used in (11.1.5), then the standard part operator can now be applied to (11.1.6) and the result is the same as $\mathtt{st}(\sum_{i=1}^{\lambda}(1/(10^\omega))$.

The process outlined in (11.1.6) is then applied to finitely many distinct intermediate propertons — those that characterize an elementary particle. The result is a properton each coordinate of which is infinitely close to the value of a numerical characterization or an infinitesimal. When the standard part operator is applied under the usual coordinatewise procedure, the coordinates are either the specific real coordinatewise characterizations or zero. Therefore, N-world formation of particles, the dense substratum field, or even gross matter may be accomplished by a ultrafinite combination of ultra-propertons that leads to the intermediate properton; followed by finite combinations of intermediate propertons that produce the N-world objects. Please note, however, that prior to application of the standard part operator such propertons retain infinitesimal nonzero coordinate characterizations in other noncharacterizing positions. (See note (1) on 60.)

We must always keep in mind the hypothesis law [9] and avoid unwarranted speculation. We do not speculate whether or not the formed particles have point-like or "spread out" properties within our space-time environment. These additional concepts may be pure catalyst type statements within some standard N-world theory and could have no significance for either the N-world or NSP-world.

With respect to field effects, the cardinality of the set of all ultra-propertons clearly implies that there can be ultrafinite combinations of ultra-propertons "located" at every "point" of any finite dimensional continuum. Thus the field effects yielded by propertons may present a completely dense continuum type of pattern within the N-world environment although from the monadic viewpoint this is not necessarily how they "appear" within the NSP-world.

There are many scenarios for quantum transitions if such occur in objective reality. The simplest is a re-ultrafinite combination of the ultra-propertons present within the different objects. However, it is also possible that this is not the case and, depending upon the preparation or scenario, the so-called "conservation" laws do not hold in the N-world.

As an example, the neutrino could be a complete fiction, only endorsed as a type of catalyst to force certain laws to hold under a particular scenario. Consider the set of sentences

$$\begin{aligned}\mathrm{G_B} = \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||}\\ &\mathrm{total|||energy|||c'{+}n'.} \mid ((i,j,n) \in\\ &\mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+) \land (1 \leq k \leq m)\}.\end{aligned} \tag{11.1.7}$$

It is claimed by many individuals that such objects as being described in $\mathrm{G_B}$ exist in objective reality. Indeed, certain well-known scenarios for a possible cosmology require, at least, one "particle" to be characterized by such a collection $\mathrm{G_B}$. By the usual method, these statements are *- transferred to

$$\begin{aligned}\mathrm{G'_B} = \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||}\\ &\mathrm{total|||energy|||c'{+}n'.} \mid ((\mathrm{i},\mathrm{j},\mathrm{n}) \in\\ &{}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+) \land (1 \leq k \leq m)\}.\end{aligned} \tag{11.1.8}$$

Hence, letting $n \in \mathbb{N}_\infty$ then various "infinite" NSP-world energies emerge from our procedures. With respect to the total energy coordinate(s), ultra-propertons may also be ultrafinitely combined to produce such possibilities. Let $\lambda = 10^{2\omega}$ [resp. $\lambda = \omega^2$] and $\omega \in \mathbb{N}_\infty$. Then

$$\sum_{n=1}^{\lambda} \frac{1}{10^\omega} = 10^\omega \text{ [resp. } \sum_{n=1}^{\lambda} \frac{1}{\omega} = \omega] \in \mathbb{N}_\infty. \tag{11.1.9}$$

Of course, these numerical characterizations are external to the N-world. Various distinct "infinite" qualities can exist rationally in the NSP-world without altering our interpretation techniques. The behavior of the infinite hypernatural numbers is very interesting when considered as a model for NSP-world behavior. A transfer of finite energy, momentum and, indeed, all other N-world characterizing quantities, back and forth, between these two worlds is clearly possible without destroying NSP-world infinite conservation concepts.

Further, observe that various intermediate propertons carrying nearstandard coordinate values could be present at nearstandard space-time coordinates, and application of the continuous and external standard part operator would produce an apparent not conserved N-world effect. These concepts will be considered anew when we discuss the Bell inequality.

Previously, ultrawords were obtained by application of certain concurrent relations. Actually, basic ultrawords exist in any elementary nonstandard superstructure model, as will now be established for the general paradigm.

Referring back to $\mathrm{G_A}$ equation (11.1.1), for some fixed k, $1 \leq k \leq m$, let $\mathrm{h}_k\colon \mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+ \to \mathrm{G_A}$ be defined as follows: $\mathrm{h}_k(i,j,n) = \mathrm{An|||elementary|||particle|||\ k'(i',j')|||with|||total|||energy|||c'{+}1/(n')}$. Since the set $F(\mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+)$ is denumerable, there exists a bijection $\mathrm{H}\colon \mathbb{N} \to F(\mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+)$. For each $1 \leq \lambda \in \mathbb{N}$ and fixed $i,\ n \in \mathbb{N}^+$, let $\mathrm{G_A}(\lambda) = \{\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||total|||energy|||c'{+}1/(n').} \mid (1 \leq \mathrm{j} \leq \lambda) \wedge (j \in \mathbb{N}^+)\}$. Let $p \in \mathbb{N}$. If $|\mathrm{H}(p)| \geq 2$, define finite $\mathrm{M}(\mathrm{h}_k[\mathrm{H}(p)]) = \{\mathrm{A_1|||and|||A_2|||and|||} \cdots \mathrm{|||and|||A}_m\}$, where $\mathrm{A_j} \in \mathrm{h}_k[\mathrm{H}(p)], m = |\mathrm{H}(p)|$. If $|\mathrm{H}(p)| \leq 1$, then define $\mathrm{M}(\mathrm{h}_k[\mathrm{H}(p)]) = \emptyset$. Let $\mathrm{M}^0 = \bigcup\{\mathrm{M}(\mathrm{h}_k[\mathrm{H}(p)]) \mid p \in \mathbb{N}\}$. Please note that the k' represents the "type" or name of the elementary particle, assuming that only finitely many different types exist, i' is reserved for other purposes, and the j' the number of such elementary particles of type k'.

Theorem 11.1.3 *For any* $i,\ n,\ \lambda \in {}^*\mathbb{N}^+$, *such that* $2 \leq \lambda$, *there exists* $w \in {}^*\mathbf{M^0} - \mathbf{G_A}$, ${}^*\mathbf{G_A}(\lambda) \subset {}^*\mathbf{S}(\{w\})$ *and if* $A \in {}^*\mathbf{G_A} - {}^*\mathbf{G_A}(\lambda)$, *then* $A \notin {}^*\mathbf{S}(\{w\})$.

Proof. Let $i,\ j,\ \lambda \in \mathbb{N}^+$ and $2 \leq \lambda$. Then there exists some $r \in \mathbb{N}$ such that $\mathrm{h}_k[\mathrm{H}(r)] = \mathrm{G_A}(\lambda)$. From the construction of M^0, there exists some $r' \in \mathbb{N}$ such that $\mathrm{w}(r') = \mathrm{An|||elementary|||particle|||k'(i',1')|||with|||total|||energy|||c'{+}1/(n').}$ $\mathrm{|||and|||An|||elementary|||particle|||k'(i',2')|||with|||total|||energy|||c'{+}1/(n').|||and|||} \cdots \mathrm{|||and|||}$ $\mathrm{An|||elementary|||particle|||k'(i',\lambda')|||with|||total|||energy|||c'{+}1/(n').} \in \mathrm{M}[\mathrm{h}_k\mathrm{H}(r')]$. Note that $\mathrm{w}(r') \notin \mathrm{G_A}$, $\mathrm{h}_k[\mathrm{H}(r')] \subset \mathrm{S}(\{\mathrm{w}(r')\})$ and if $\mathrm{A} \in \mathrm{G_A} - \mathrm{h}_k[\mathrm{H}(r)]$, then $\mathrm{A} \notin \mathrm{S}(\{\mathrm{w(r')}\})$. The result follows by our embedding and *-transfer. ∎

The ultrawords utilized to describe various propertons, whether obtained as in Theorem 11.1.3 or by concurrent relations, are called *ultramixtures* due to their applications. The ultrafinite choice operator $\mathbf{C_1}$ can select them, prior to application of ${}^*\mathbf{S}$. Moreover, application of the ultrafinite combination operator entails a specific intermediate properton with the appropriate nearstandard coordinate characterizations. Please notice that the same type of sentence collections may be employed to infinitesimalize all other quantities, although the sentences need not have meaning for certain popular N-world theories. Simply because substitution of the word "charge" for "energy" in the above sentences $\mathrm{G_A}$ does not yield a particular modern theory description, it does yield the infinitesimal charge concept prevalent in many older classical theories.

Using such altered $\mathrm{G_A}$ statements, one shows that there does exist ultramixtures w_i for each intermediate properton and, thus, a single ultimate ultramixture w such that ${}^*\mathbf{S}(\{w_i\}) \subset {}^*\mathbf{S}(\{w\})$. Each particle or elementary particle may, thus, be assumed to originate from w through application of the ultralogic ${}^*\mathbf{S}$.

11.1.2 More on Propertons

The general process for construction of all N-world fundamental entities from propertons can be improved upon or achieved in an alternate fashion. One of the basic assumptions of subatomic physics is that in the Natural-world two fundamental subatomic objects, such as two electrons, cannot be differentiated one from another by any of its Natural-world properties. One of the conclusions of what comes next is that in the NSP-world this need not be the case. Of course, this can also be considered but an auxiliary result and need have no applications. At a particular instant of (universal) time, it is possible to associate with each entity a distinct "name" or identifier through properton construction. This is done through application of the properton naming coordinate a_1. As will be shown, the concept of independent *-finite coordinate summation followed by n-tuple vector addition can be accomplished by means of a simple linear transformation. However, by doing so, the concept of the *-finite combinations or the gathering together of propertons as a NSP-world physical-like process is suppressed. Further, a simple method to identify each N-world entity or Natural-system is not apparent. Thus, we first keep the above two processes so as to adjoin to each entity constructed an appropriate identifier.

There is a coding i used in the original foundations, that codes each member of $\mathcal{W}'$ via the natural numbers. (Herrmann, 1979-1993). This coding can be dropped and has no effect upon and the results in the new foundations that include a general language such as $\mathcal{W}'$ as a subset of the ground set. This embedding is now removed as allowed by the constructions in Herrmann (1979-1993). Further, a general language $\mathcal{W}'$ is now considered as a subset of the ground set for the standard model $\mathcal{M}_1$. As mentioned in Herrmann (1997-93), the members of $\mathcal{W}'$ can be expressed in a different color than all other symbols used. (Apparently, Robinson (1963) was the first to use only such a ground set, (i.e. not using $\mathcal{E}$). He differentiates between members of $\mathcal{W}'$ by simply stating that they are individuals and are different relative to all the symbols that appear in another set.)

There is a bijection $w\colon \mathcal{W}' \to \mathcal{E}$, such that, for $a \in \mathcal{W}'$, $w(a) = [g]$. The only difference between whether i is included or not is that for an $[k] \in \mathcal{E}$, the range of the partial sequence k is a subset of $i[\mathcal{W}']$, a set of natural numbers, or the range is a subset of $\mathcal{W}'$, respectively. It is but a matter of "style" whether i is included or not. There are two special unique members in each $[g], f_0,\ f_m$. Intuitively, via the Markov join operator, $f_m \in [g],\ f_m \in {\mathcal{W}'}^{[0,m]}$ and the "ordered" $a = f_m(m)\cdots f_m(0)$, and each $f_m(j) \in \mathcal{A}'$ the extended alphabet. Then $f_0 \in [g]$ and $f_0(0) = a$. Of course, relative to a physical or non-physical world, all such mathematically modeled objects but "represent" entities or processes.

Definition 1.1. For fixed even $K > 2$ and $n \in \mathbb{N}'$, consider the sets of K+2-tuples $\mathrm{C}_n = \{(\mathrm{h}(j), 1, -1/10^n, 1/10^n, \ldots, -1/10^n, 1/10^n) \mid (i \in \mathbb{N}) \wedge (n \in \mathbb{N})\}$.

From definition 1.1, via *-transform, we have the hyperfinite set

$$^*C = \{(\ ^*h(j), 1, -1/10^n, 1/10^n, \ldots, -1/10^n, 1/10^n) \mid (j \in {^*\mathbb{N}}) \wedge (n \in {^*\mathbb{N}})\}.$$

Let $\omega \in {^*\mathbb{N}'} - \mathbb{N}'$.

Definition 1.2, Ultra-propertons. The set of all ultra-propertons is (represented by) the hyperfinite $C = \{(\ ^*h(j), 1, -1/10^\omega, 1/10^\omega, \ldots, -1/10^\omega, 1/10^\omega) \mid (j \in {^*\mathbb{N}})\}$.

For Definition 1.2, it is assumed that there is no more than K physical or physical-like numerical or coded descriptive characteristics for the any elementary entity.

Let $r_1 \in \mathbb{R}$. By Theorem 11.1.1 in Herrmann (1979-1993), there is a $\lambda_1 \in \mathbb{N}_\infty$ such that $\lambda_1/10^\omega \in \mu(|r|)$. Hence, $\mathtt{st}((\lambda_1/10^\omega)) = |r|$. Then there are $K,\ \lambda_i,\ i \in [1, K]$ that yield the K characteristics. For an elementary entity e_j, some characteristics can be 0, meaning that the measure has value 0. Throughout the combining processes, if a coordinate retains its infinitesimal value $\pm 1/10^\omega$, this indicates that the characteristic has no meaning for e_j. In order to indicate these differences, any characteristic that has measure 0 is obtained from a combination of two ultra-propertons. The standard part physical realization operator $\mathtt{st}(\cdot)$ is only applied to coordinates of the intermediate properton representations with the form $\pm\lambda/10^\omega$, where $\lambda \geq 2$.

There are other characteristics such as spin, where the 0 takes on a different meaning. However, such coding is rather arbitrary and can be replaced with non-zero numbers or non-zero codings for the characteristics so as to not confuse them with a 0 measurement. For the needed intermediate properton e_1, with a third coordinate characteristic under independent coordinate addition, the hyperfinite set of ultra-propertons $\{(\ ^*h(j), 1, -1/10^\omega, \ldots, 1/10^\omega) \mid j \in [1, \lambda_1]\}$ is employed. Hence, the first intermediate properton is $(\Pi_1^{\lambda_1}, \lambda_1, -\lambda_1/10^\omega, 1/10^\omega, \ldots, 1/10^\omega)$. For a forth coordinate intermediate properton for value r_2, consider $\{(\ ^*h(i), \lambda_2, -1/10^\omega, \lambda_2/10^\omega, \ldots, 1/10^\omega) \mid i \in [\lambda_1+1, \lambda_1+\lambda_2]\}$. Continue these definition for each member of $[1, K]$. Thus the entire collection of ultra-propertons used to obtain one of the e_1 entities is $\lambda_1 + \cdots + \lambda_K = \delta_1 \in \mathbb{N}_\infty$.

It is assumed that there are a nonempty countable (i.e non-zero finite or denumerable) collection of $\{e_i\}$ needed. Thus there is a non-zero finite or denumerable set $\{\delta_i\}$ and in the finite case, consider $\sum \delta_i \in \mathbb{N}_\infty$. Next consider $\{\delta_i \mid i \in \mathbb{N}'\}$. Then $\{\delta_i \mid i \in \mathbb{N}'\} \subset {}^*\mathbb{N}$. The $|\{\delta_i \mid i \in \mathbb{N}'\}| < |\mathcal{M}_1|^+$. Hence, there is a $\gamma_1 \in \mathbb{N}_\infty$ such that $\{\delta_i \mid i \in \mathbb{N}'\} \subset [1, \gamma_1]$ by application of Corollary 1.1.1 and Theorem 1.2. Thus, in both cases, there is a $\Gamma_1 \in \mathbb{N}_\infty$ such that $\{\delta_i\} \subset [1, \Gamma_1]$. This shows that there are "enough" ultra-propertons to produce the set $\{e_i\}$. For another type of elementary particle, simply repeat this for the identifiers $h(j),\ j > \Gamma_1]$. Then continue by induction.

For this application, it appears unnecessary to consider more than H, where $1 \leq H \in \mathbb{N}$, different types of elementary entities. The set of ultra-propertons $\{(\ ^*h(j), 1, -1/10^\omega, \ldots, 1/10^\omega) \mid j \in {}^*\mathbb{N}'\} = C$ is an internal set and as such the hyperfinite operator ${}^*\mathcal{F}$ is defined for it. For properton generation, a universe can be considered as a collection of physical-systems. Hence application of a finite (> 0) iteration ${}^*\mathcal{F}^i$ to C yields $\mathcal{C} = \bigcup\{\ {}^*\mathcal{F}^i(C) \mid (1 \leq i \leq n) \land (i \in \mathbb{N})\}$, an internal collection that is sufficient to generate the physical-systems for any of the presently considered cosmologies. To accommodate the formation of the physical-like systems, infinite hyperfinite set X of internal sets that is disjoint from $\bigcup\{\ {}^*\mathcal{F}^i(C) \mid (0 \leq i \leq n) \land (i \in \mathbb{N})\}$ is adjoined to $\bigcup\{\ {}^*\mathcal{F}^i(C) \mid (0 \leq i \leq n) \land (i \in \mathbb{N})\}$ Then $\Pi^+ = \mathcal{C} \cup X$.

Rather than the $\mathrm{f}^q(i,j)$ being a general description, one considers instructions or rules $\mathrm{I}^q(i,j)$ - a nonempty finite subset of $\mathcal{W}'$, which is equivalent to a single word in $\mathcal{W}'$. These sets of instructions - instruction-sets - (also called instruction-information) are indexed in the same way as the general descriptions and determine the *instruction paradigm* $\mathcal{I}_q$. Indeed, there is an injection H on d_q onto $\mathcal{I}_q$, where $\mathrm{H}(\mathrm{f}^q(i,j)) = \mathrm{I}^q(i,j)$ and (i,j) varies over the same set of integers and natural numbers, respectively. (See appendix.) There is one instruction paradigm for each pre-designed universe and there can be a vast collection of such universes. Rather than simply applying this bijection as a means to reproduce each of the instruction paradigm results, the instruction paradigm is analyzed directly.

As symbol strings, the set $\dagger\{\mathrm{There}|||\mathrm{are}|||\mathrm{n}'|||\mathrm{ultra} - \mathrm{propertons}|||\mathrm{combined}|||\mathrm{to}||| \mathrm{produce}|||\mathrm{an}|||\mathrm{intermediate}|||\mathrm{properton.} \mid \mathrm{n} \in \mathbb{N}\}$ is a member of $\mathcal{N}$. But the terms "ultra-properton" and "properton" have no physical meanings within the physical world. Further, it corresponds under w to a specific subset of $\mathcal{E}$. Using the method of Theorem 9.3.1 in Herrmann (1979-93) and *-transform, this set corresponds to a subset of ${}^*\mathcal{W}'$ as well as a subset of ${}^*\mathcal{E} - \mathcal{E}$, where the n' now varies over members of $\mathbb{N}$ for one subset and $\mathbb{N}_\infty$ for the other. But the language $\mathcal{W}'$ does not have alphabet symbols that correspond to members of $\mathbb{N}_\infty$. For specific members of $\mathbb{N}_\infty$, representative symbols, constructed or otherwise, that are not members $\mathcal{W}'$ are employed. There are objects in ${}^*\mathcal{W} - \mathcal{W}$ that do correspond to symbols in the extend alphabet ${}^*\mathcal{A}'$. One such object is denoted in the following display by the λ. This leads to a specific ordered string of symbols via the corresponding member of ${}^*\mathcal{E}$ that is intuitively represented by

$$\dagger \mathrm{There}|||\mathrm{are}|||\lambda|||\mathrm{ultra-propertons}|||\mathrm{combined}|||\mathrm{to}|||$$

$$\mathrm{produce}|||\mathrm{an}|||\mathrm{intermediate}|||\mathrm{properton.}$$

Such † *instructions have “interpretations” in terms of the GGU-model language as do other members of ${}^*\mathcal{M}$ or ${}^*\mathcal{M}_1$. They can have additional symbol strings taken from $\mathcal{W}'$ that have no meanings until interpreted for the GGU-model.

Each physical-system is actually a physical-like system since their construction includes *instructions for, at present, unknowable processes or “things,” as represented by the X, that seem to “force” these combinations to occur. Notice that symbol-strings, diagrams, images and sensor information represented by a standard language $\mathcal{W}'$ are comprehensible only when they carry an additional component - meanings. ”Meanings” are understood by the mind and cannot lead to mere circular thinking.

For human endeavors, there is a five-step process. (1) A standard meaningful instruction-set. (2) The instructions are mentally comprehended. (3) This mental comprehension is transmitted, via electro-chemical actions, to other human physical locations. (4) At these physical locations actions are performed. (5) These actions produce a physical entity that corresponds to the original instruction-set. Errors can occur along this entire path. If these processes are performed in an errorless manner and the physical entity produced does not correspond to a desired physical object, then this alone cannot change the instruction-set. The instruction-set (1) would need to be altered in a meaningful way as required by (2). This alteration is done by an intelligent physical entity. For the GGU-model, a standard meaningful instruction-set contains operative statements.

Aside from such human endeavors, Nature neither displays physical laws nor numerical parameters. We display such laws and parameter values as representations for behavior we cannot otherwise comprehend. By definition, such objects as quantum fields or strings are not displayed by Nature as independent agents. Again they are used as representations for behavior we cannot, at present, otherwise comprehend. Hence, distinct from human endeavors, steps 1 - 5 represent behavior we cannot, at present, otherwise comprehend.

From the modeling viewpoint, processes within the substratum are non-physical. Thus, step (1) is non-physical and steps (2) and (3) are consider as statements relative to a non-physical medium and a non-physical mode of transmission, respectively. For each universe-wide frozen-frame, step (4) corresponds to a subsets of $\mathcal{C}$ - the substratum info-fields. Step (5) is the application of the $\mathtt{st}(\cdot)$ operator and yields that physical products of the entire process. In order to be operative, the above displayed *instruction as well as similar ones have non-physical components.

The process of applying the standard part operator does not erase the original coordinates of the constructed objects. The process only yields the physical-world codes. Such alterations of the fundamental subatomic or field entities is modeled by vector space subtraction and this would be followed by independent *-finite subtraction. This represents a “breaking apart” of the fundamental entity into its original ultra-properton constituents. The original identifiers are restored. An altered constituent is then obtained by repeating the construction process. This can be used to eliminate the virtual particle or process concept within reality models. The models that need such concepts to predict behavior are now just that “models” for predicted behavior and would not require the virtual “stuff” to exist in reality.

As mentioned, the formation of fundamental entities from ultra-propertons through the process of the intermediate properton formation can be modeled by means of a very simply (diagonal styled) *-continuous linear transformation. Let a fundamental entity have the required finitely many coordinate numerical characteristics as discussed previously and let nonempty finite $A \subset {}^*\mathbb{N}$ be the specific coordinate names. Then there is a finite set of hypernatural numbers $\{\lambda_i \mid I \in A\}$ that represent the number of summands in the respective *-finite summation process. (The process that yields the intermediate propertons.) Consider an $n \times n$ hypermatrix with the diagonal elements $\lambda_i = b_{ii},\ i \in A$, and $b_{jj} = 1,\ j \in ({}^*\mathbb{N} - A),\ j \leq n$ and $b_{ij} = 0$ otherwise. Then letting (a_j) be a coordinate representation for an ultra-properton, consider $(b_{ij})(a_j)^{\mathrm{T}}$. This hypermatrix represents an internal function that will take a single *representation* for an ultra-properton and will yield, after application of the standard part operator to the appropriate coordinate values (or all of them if one wishes), a *representation* for a fundamental entity. BUT note that identifier coordinate would not be that of the intermediate properton. Unless we use the sum of the λ_is in the second diagonal place, the counting coordinate would not be the appropriate value. Of course, the matrix inverse would yield a representation of an original ultra-properton with its identifier. Thus using either method, the identifier would be lost to the N-world. But using one of the favorite simple mathematical models, the matrix approach, the fundamental entity identifier would be lost to the NSP-world as well. This shows how two different mathematical procedures can lead to equivalent N-world results, but yield considerably different NSP-world ramifications.

The next step in a properton re-formulation of particle physics would be to consider specific NSP-world predictions for what is claimed are random or, at least, probabilistic in character actual observed events. In this regard, I mention the known fact that a probably density functions behavior can be predicted by a specific well-defined sequence. For example, it is known that there is a real number x and a specifically defined sequence $x2^n$ such that if one takes the fractional part of each $x2^n$, less than 1/2 and correspond it to H, and the fractional part greater or equal to 1/2 and correspond it to a T, then the predictive results will pass every statistical test that implies that H and T are randomly obtained by a sequence of H = heads, T = tails coin tosses. Clearly, this process, as far as the sequence $x2^n$ is concerned is a predictable process and contradicts scientific randomness. But the specific numbers x need not be known by entities within the Natural universe.

11.2 An Ultimate GGU-model Conclusion.

In the book "Ultralogics and More" other properton properties are discussed. These include the ultraenergetic and ultrafast propertons. The ultrafast propertons are of significance in that they can be used to provide seemingly instantaneous informational transmissions throughout the Natural universe. This is significant for the ultralogic, ultraword (and ultra-logic-logic) generation of a universe. Clearly the research presented in this monograph is not complete but is only designed to present the beginning concepts in what is hoped will be a continuing research activity. One important question is relative to the fact that an ultimate ultraword such as w' generates a preselected or theory generated ideal universe. How can this be made to correspond to the actual universe in which we live where each Natural-system is perturbed or altered within certain limits dictated by Natural law from this ideal case? Such alterations can also be produced by partially independent agencies such as biological entities. There are different ways to attack this problem but the existence of the UN-events would require that speculation be restrained.

One method is slightly similar to the Everett-Wheeler-Graham many-worlds interpretation (parallel universes) but is much less esoteric in character. One can consider countably many w'_t from

Corollary 7.3.4.1 each one containing the allowable alterations in Natural-system behavior starting from the moment t. Denote the corresponding set of universes by $\mathcal{U}$. At any instant of substratum time t, all the universes but one are "covirtual" universes. This does not necessary mean that they actually exist in some type of NSP-world reality. This simply means that they exist potentially. When any one or more Natural-systems is perturbed then the UN-events react first so to speak. Information relative to this reaction is transmitted within $\{w'_t\}$ by means of ultrafast propertons. This information causes a w'_{U_t} to be selected such that $U_t \subset {}^*\mathbf{S}(\{w'_{U_t}\}),\ U_t \in \mathcal{U}$. For the "next" instant, this generated U_t becomes the Natural universe objective reality. Now technically there would be a "next" instant since there would be an NSP-world time interval over which all of the Natural-systems would have a fixed frozen segment segments that would not be altered. The only possible alterations would be in the UN-events. This process then continues throughout all of Natural universe time.

There is a second approach that can solve this perturbation problem but requires a complete restructuring of ultraword generation. This method is similar to the concept of "parallel" logic. Time in this case is modeled as previously done but on interval $(-\infty, +\infty)$. At a particular observer beginning instant t, a time slice in the mathematical sense is obtained. This slice will yield a frozen segment describing a Natural event for each of the present Natural-systems. This will lead as in Chapter 7 to an ultraword x that will by means of ${}^*\mathbf{S}$ generate this slice. Let $\mathcal{NA}$ denote the set of all potential Natural-systems. This means that some of these at the instant t contain only frozen segments from ${}^*\mathbf{T}_i - \mathbf{T}_i$.

Assume that $\mathcal{NA}$ is at the least denumerable. Then each x_t (the ultraword associated with the instant t) also contains UN-events. Again the alterations in Natural-system behavior within the limits of Natural law lead to changes in the frozen segment descriptions for the very "next" Natural event. These changes are incorporated within the NSP-world time interval into the "next" ultraword x_{t_1}. Application of ${}^*\mathbf{S}$ to x_{t_1} now yields the "next" slice. This process continues throughout all of the Natural universe time frame. Of course the "time" being referred to here is not N-world measured time but the time concept previously discussed that is concerned with "before and after."

The first of the above models appears to be the most significant and the one that is probably the closest to objective reality. This portion of this model leads to the moment-to-moment re-generation of our universe since its behavior most certainly is perturbed moment-to-moment from the ideal.

As asked previously, is there something within Nature that "forces" physical behavior within our universe to follow expressed laws as well as to satisfy expressed parameter values? GGU-model descriptions represent non-physical substratum entities and behavior that provide an answer to this question.

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(1) For propertons, only two possible intrinsic properties for elementary particle formation are here considered. Assuming that there are such things as particles or elementary particles, then they would be differentiated one from the other by their intrinsic properties that are encoded within properton coordinates. When there are particle interactions, these intrinsic properties can be altered or even changed to extrinsic properties. How the alteration from intrinsic to extrinsic occurs probable cannot be known since it most likely is an ultranatural event. For new results on properton formations, see http://arxiv.org/abs/quant-ph/9909078

(2) To conceive of subparticles properly, quantum theory is viewed as an approximation. Moreover, in terms of physically determined units, the numerical characteristics produced by applications of the standard part operator are considered as exact.

Appendix-Ultra-logic-systems

1. Logic-System Generation for Instructions

As is customary, the nonstandard model used in all of the articles on the GGU-model is a polysaturated polyenlargements (Lobe and Wolff, 2000; Stroyan and Bayod, 1986). In this paper, $q = 1, 2, 3, 4$. These numbers denote the four primitive-time intervals (Herrmann 2006) employed for the GGU-model. The ultraword approach to generate a universe is replaced with an ultra-logic-system. This is a hyperfinite logic-system where, after application of the extended logic-system algorithm, generates each member of the hyperfinite instruction paradigm d_x^q in the proper $\leq_{d_x^q}$ order such that $\mathbf{d}_q \subset d_x^q \subset {}^*\mathbf{d}_q$, where $q = 1, 2, 3, 4$ and $x = \lambda, \nu\lambda, \mu\lambda, \nu\gamma\lambda$ respectively. Finally, in this article, the term "properton" was previously used. To prevent incorrect mental images as to models for propertons, the term "properton" replaces the term "subparticle." Without visualizing, a properton is an entity characterized only by a list of properties.

The primitive entity that yields physical reality for any GGU-model generated universe is dense collection of ultra-propertons. When first conceived this author had not investigated quantum field theory and did not base proprtins upon any quantum theoretic approach. All of the GGU-model entities and processes can be considered as existing in a **background universe** or **substratum world**. This world can be considered as a physical-like world, where the rules that govern universe formation are distinct from those processes and rules that govern the development of <u>any</u> physical universe. They are simple rules that only refer to counting. This substratum world is also interpreted philosophically in other ways.

If necessary for a specific physical theory, any continuity requirement is satisfied by the properton field (Herrmann, 1983, 1989). For our universe, a collection of propertons has been shown to be closely associated with relativistic effects (Herrmann, 2003). No other known primitive entities, such as superstrings, will have any effect upon the application of propertons as the primitive entities that generate a universe. The processes used to obtain particles and all other physical entities from ultra-propertons need not correspond to the rules of quantum field theory or any additional rules like how quarks combine to form particles.

For our universe, quantum field theory contains descriptions (rules or instructions) that produce such particles from immaterial fields. Such fields are quantum mechanical systems and, when represented, have various degrees of freedom. These are but parameters that contribute to the overall state of the system. For various particles, parameters for physical measures or states are the characterizing features of propertons. The physical appearance and disappearance of particles are trivial applications of properton processes. For quantum field theory, one has the "creation" and "annihilation" operators that mathematically yield the same results.

For the GGU-model, quantum theory does not produce steps in a development since the method of production must be universe and physical law independent. For our universe, the development "satisfies" the predictions of accepted physical theories. I personally consider quantum theory as mostly a product of human imagination that predicts behavior, behavior that we cannot otherwise comprehend. That is, it is a model that mimics.

The GGU-model can be based upon observable human behavior and the mathematics predicts, for our universe, behavior that satisfies the behavior predicted by accepted physical theories. There is a vast amount of evidence for the predicted GGU-model processes. Whether such processes exist in some sort of reality is a philosophic choice. One can make this choice based upon various factors.

One can choose to accept properton existence based upon the same philosophy expressed by those that accept that entities postulated in quantum field and particle theory exist.

The concept of instructions or rules is generalized to instructions that yield a physical reality from combinations of propertons. They are substratum laws. (So as not to confuse these with physical laws, they are called instructions. Further, in what follows, the events that correspond to each $\mathrm{f}^q(i,j)$ are denoted by $\mathrm{E}^q(i,j)$.) This does <u>not</u> mean that the rules used in quantum theory (QT) actually yield each $\mathrm{E}^q(i,j)$. As mentioned, what this signifies is that the QT rules are verified via the production of event sequences that yield our universe. For the GGU-model, the physical realization of each $\mathrm{f}^q(i,j)$ is not the result of any of these physical theories. These theories are but verified by each realized $\mathrm{f}^q(i,j)$ and they allow us to predict what behavior occurred in or will occur within other realized $\mathrm{f}^q(p,k)$. For the GGU-model, the "instructions" are rather simple ones that lead to all the characteristics that allow one to identify any material entity for any of the presently known cosmologies.

Rather than the $\mathrm{f}^q(i,j)$ being a general description, one considers instructions or rules $\mathrm{I}^q(i,j)$ - a nonempty finite subset of L, which is equivalent to a single word in L. These sets of instructions - instruction-sets - (also called instruction-information) are also indexed in the same way as the general descriptions and determine the *instruction paradigm* $\mathcal{I}_q$. Indeed, there is an injection H on d_q onto $\mathcal{I}_q$, where $\mathrm{H}(\mathrm{f}^q(i,j)) = \mathrm{I}^q(i,j)$ and (i,j) varies over the same set of integers and natural numbers. There is one instruction paradigm for each pre-designed universe and there can be a vast collection of such universes. Rather than simply applying this bijection as a means to reproduce each of the instruction paradigm results from the developmental paradigm results, what follows is a duplicate of these results and how they are obtained in terms of instruction paradigm notation.

Relative to the GGU-model and generation of a universe, a hyperfinite ${}^*\mathbf{I}^q(i,j)$ yields a universe-wide frozen-frame. Each instruction $x \in {}^*\mathbf{I}^q(i,j)$, yields a physical or physical-like system. The physical-systems are disjoint. Each collection of ultra-propertons that yields a specific physical-system is distinct from the set of ultra-propertons that yields any other physical-system. Hence, each physical-system within a universe-wide frozen-frame has a distinct identifier via the collection of all of the identifiers for the ultra-propertons or the intermediate propertons employed to produce the physical-system.

2. Logic-System Generation for the Type-1 Interval.

The notation in all that follows is from Herrmann (2006). Notice that there are two different t sequence notations. One t is in the informal world, while another t is in the formal standard superstructure. These two sequence are, of course, consider as equivalent since the set of objects that informally yield the informal t are also formally present within the standard superstructure. The informal composition $\mathrm{f}^q = \mathrm{I}^q \circ \mathrm{t}^q$, when embedded relative to $\mathcal{E}$ is denoted by $\mathbf{f}^q = \mathbf{I}^q \circ t^q$ since the t^q is not embedded relative to $\mathcal{E}$ and it merely generates a rational number sequence for the embedded informal paradigm. Usually, these different notations are eliminated and only the math-italics font is employed. This is the customary practice throughout Herrmann (1979 - 1993). Notation for informal natural, rational and real numbers, if applicable, is usually the same for the informal and more formal superstructure objects. Each $t^q(i,j)$ is a rational number. Each $\mathbf{f}(i,j)$ is a nonempty instruction-set.

Each member of $\mathcal{I}_q$ is now considered as determined by a function defined on a set R_q of rational numbers, $\mathbf{Q}$. The members of R_q carry the restricted rational simple order and the order $\leq_{\mathcal{I}^q}$ for the members of $\mathcal{I}_q$ (the lexicographic order) is order isomorphic to R_q in the obvious way.

Each interval partition is of the form $[c_i, c_{i+1})$ (with a closed interval in two cases), where $i \in \mathbf{Z}$ and $\mathbf{Z}$ is the set of integers, and $t^q(i,0) = i,\ t^q(i+1,0) = i+1$. Then each member of (c_i, c_{i+1}) is a defined rational number $t^q(i,j)$, where $i < j < i+1$. For example, consider $[c_2, c_3)$. Then $t^q(2,1) = 3 - 1/2,\ t^q(2,2) = 3 - 1/4,\ t^q(2,3) = 3 - 1/8$, then, in general, $t^q(2,j) = 3 - 1/2^j$. Hence, $\mathrm{f}^q(2,0) <_{\mathcal{I}^q} \mathrm{f}^q(2,1) <_{\mathcal{I}^q} \mathrm{f}^q(2,2) <_{\mathcal{I}^q} \cdots <_{\mathcal{I}^q} \mathrm{f}(3,0)$. (The order $\leq_{\mathcal{I}^q}$ is lexicographic and is isomorphic to the rational number order for a specific set of rational numbers.)

Let $\mathcal{I}_1$ be the standard instruction paradigm. An instruction paradigm is defined mathematically in the exact same manner as that of the developmental paradigm in Herrmann (2006) and is equivalent to the range of a sequence $\mathrm{g}'\colon \mathbb{N} \to \mathcal{P}(\mathcal{W}')$, where $\mathcal{W}'$ is our denumerable general language. The first case illustrated for the GGU-model is for a developing universe starting with a frozen segments (frame) instruction-set $\mathrm{g}'(0)$. For the other three GGU-model cases, this sequence is appropriately modified. In all cases, the $(\mathrm{f}^q(i,j), \mathrm{f}^q(p,k))$ is equivalent to "If $\mathrm{f}^q(i,j)$, then $\mathrm{f}^q(p,k)$). This notation will be simplified later.

For the type-1 case $[0,b],\ b > 0$, as indicated above, a denumerable instruction paradigm displays a refined form. For $1 < m \in \mathbb{N},\ \mathcal{I}_1 = \{\mathrm{f}^1(i,j) \mid (0 \leq i \leq m) \wedge (i \in \mathbf{Z}) \wedge (j \in \mathbb{N})$. (For each of the types, $\mathrm{f}(i,j) \in \mathcal{W}'$. Using $\mathcal{I}_1$, consider the following logic-system.

Due to the simplicity and special nature of the logic-systems used, a simplified algorithm is employed. The basic logic-system algorithm is re-defined for sets of two distinct objects $\{\mathrm{A}, \mathrm{B}\}$. If a deduction yields C and C is a member of $\{\mathrm{A}, \mathrm{B}\}$, then the "other" member is a deduction. Hence, if A is deduced, then from $\{\mathrm{A}, \mathrm{B}\}$, B is deduced. This can be written as $\{\mathrm{A}, \mathrm{B}\} - \{\mathrm{A}\}$ is deduced. In general, this approach is only valid for these special collections of two element sets. This process mimics the proposition-logic modus ponens rule of inference $\{(\mathrm{X} \to \mathrm{Y}, \mathrm{X}, \mathrm{Y}) \mid \mathrm{X},\ \mathrm{Y}$ are propositions$\}$. However, for both logic-systems only one member of any two element set is deducible.

Definition 2.1 Let $i \in \mathbf{Z}$. For each $n \in \mathbb{N}$, let $\mathrm{k}^1_i(n) = \{\{\mathrm{f}^1(i,j), \mathrm{f}^1(i,j+1)\} \mid (0 \leq j \leq n-1) \wedge (j \in \mathbb{N})\}$, $\mathrm{K}^1(n) = \bigcup\{\mathrm{k}^1_i(n) \mid (0 \leq i < m) \wedge (i \in \mathbf{Z})\}$. Finally, let finite $\Lambda^1(n) = \{\mathrm{f}^1(0,0)\} \cup \mathrm{K}^1(n) \cup \{\{\mathrm{f}^1(p-1,n), \mathrm{f}^1(p,0)\} \mid (0 < p \leq m) \wedge (p \in \mathbf{Z})\}$ and $\mathcal{L}^1 = \{\Lambda^1(x) \mid x \in \mathbb{N}\}$. The set $\{\{\mathrm{f}^1(p-1,n), \mathrm{f}^1(p,0)\} \mid (0 < p \leq m) \wedge (p \in \mathbf{Z})\}$ is called the "jump elements." Also, each $\Lambda^1(n)$ is a finite set.

In general, members in $\mathcal{L}^q$ can be characterized by a first-order sentence. When the deduction algorithm is applied to $\Lambda^1(n)$ the result is an ordered set of words from $\mathcal{W}'$ - the ordered instruction paradigm. In accordance with the juxtaposition join operator that yields words in $\mathcal{W}'$, this ordered instruction paradigm is a word in $\mathcal{W}'\mathrm{L}$. It can be obtained using the spacing symbol where each member of this paradigm is considered a sentence. For a multi-universe theory, each such universe is a portion of each of the original members of the instruction paradigm.

In order to make the notation as simple as possible for the next construction, notice that $\mathcal{L}^1$ is denumerable. Let $\mathbb{N} - \{0\} = \mathbb{N}'$. Thus, there is a bijection $\mathrm{D}^1\colon \mathbb{N}' \to \mathcal{L}^1$. We use the subscript notation for this bijection. Thus, consider $\mathcal{L}^1 = \{\mathrm{D}^1_i \mid i \in \mathbb{N}'\}$. For each $n \in \mathbb{N}'$, define $\mathrm{M}^1_n = \{\{\mathrm{D}^1_1, \ldots, \mathrm{D}^1_n\}\}$. Let $\mathcal{M}^1 = \{M^1_n \mid n \in \mathbb{N}'\}$. The set $\mathrm{M}^1_n = \{\{\mathrm{D}^1_1, \ldots, \mathrm{D}^1_n\}\}$, as before, can be considered as a single word-like object.

(There are a few typographic errors in Herrmann (2006) and (2006a). For example, in Theorem 4.1, $m > 0$ should read $m > 1$, and ${}^*\mathbf{D}$, should read ${}^*\mathbf{D}_1$. In Herrmann (2006a), page 12, in the first (4), the $\nu \in {}^*\mathbf{Z}^{\geq 0} - \mathbf{Z}$ should be replaced with $\nu \in {}^*\mathbf{Z}^{\leq 0} - \mathbf{Z}$, $\gamma \in {}^*\mathbf{Z}^{\leq 0} - \mathbf{Z}$ should be replaced with $\gamma \in {}^*\mathbf{Z}^{\geq 0} - \mathbf{Z}$.)

A finite consequence operator S is defined in Herrmann (1979 - 1993, p. 65). However, a new simplified logic-system $\mathcal{S}^q,\ q = 1, 2, 3, 4$ is defined. When a logic-system is applied, it generates a specific finite consequence operator. It is the logic-system algorithm that does this. In this article, this algorithm is explicitly noted since only logic-systems are used. In general, logic-systems are stated in terms of metamathematics n-tuples. If a set $\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \ldots, \mathrm{D}\}$ is used as an hypothesis, then it is word-like since the objects the logical deduction models via the algorithm yields words or word-like objects.

Define $\mathcal{M}^q,\ q = 2, 3, 4$, in the same manner as $\mathcal{M}^1$, from members of $\mathcal{L}^q$. For each $\mathrm{G}^q \in \mathcal{M}^q$, there exists a unique $n \in \mathbb{N}'$ such that $\mathrm{G}^q \in \mathrm{M}^q_n$. This $\mathrm{G}^q = \{\mathrm{D}^q_1, \ldots, \mathrm{D}^q_n\},\ \mathrm{D}^q_i \in \mathcal{L}^q,\ 1 \leq i \leq n$.

Define the logic-system that generates S^q as $\mathcal{S}^q = \{\{x, y\} \mid (\exists n(n \in \mathbb{N}')) \wedge (x \in \mathrm{M}^q_n) \wedge (y \in \mathcal{L}^q) \wedge (y \in x)\}$. (This definition can be further described in order to characterize the doubleton set notion and can include all necessary bounds for the quantifiers.) Further, under the simplification used here, each member of $\mathcal{S}^q$ is a propositional tautology. Notice that M^q is a function with values a singleton set containing an n-set (i.e. a set of "n" members).

Usually, such a logic-system would use ordered pairs to model the rules of inference. Within these rules, finite conjunctions are displayed as first coordinates via n-sets. Again the simplified doubleton-set approach is used here, where one of these sets is $\{\{D_1\}, D_1\}$.

Hypotheses are considered as members of a set (a 1-ary relation), when part of a logic-system. They are, usually, considered as a list of the members of this set. In general, a logic-system, when considered as an operator, is defined on subsets of the language employed.

From the definitions employed for the logic-systems used here, the properties of the logic-system algorithm $\mathcal{A}$ can be explicitly described in set-theoretic notation. For these applications, $\mathcal{A}$ is a function defined on various defined logic-systems and a set of hypotheses. For example, the entire set of deductions or the order in which the deductions are made, among a few other characteristics. In our application to a logic-system, the notation used signifies all of the "deduced" results the algorithm produces when the logic-system is applied to a set of hypotheses. This yields the same results as a corresponding finite consequence operator. What the notation indicates is that the finite consequence operator is being displayed in a more refined and explicit manner. Hence, the algorithm and its relation to the logic-system can be embedded into the formal structure via formalizable characteristics.

When the application characteristics are *-transferred, then the notation ${}^*\underline{\mathcal{A}}$ is employed. The process of applying the algorithm to the logic-system $\mathcal{S}^q$, that is applied it to a set of hypotheses Y, is denoted by $\mathcal{A}((\mathcal{S}^q, \mathrm{Y}))$. Hence, $\mathcal{A}$ is defined upon a set of ordered pairs. The result of $\mathcal{A}((\mathcal{S}^q, \mathrm{Y}))$ is a set. An additional step can be included for this specific algorithm, where Y is removed. When this is done the algorithm is denoted by $\mathcal{A}'$. The necessary informally and, hence, formally described properties are specifically displayed. In general, the q notion is not included as part of the $\mathcal{A}$ notation unless confusion would result.

For the denumerable set $\mathcal{L}^1$, notice that for any $\Lambda^1(k),\ k \in \mathbb{N}$ there exists an $k' \in \mathbb{N}$ and $\mathrm{X}^1_{k'} \in \mathrm{M}^1_{k'}$, such that $\Lambda^1(k) \in \mathcal{A}'((\mathcal{S}^1, \{\mathrm{X}^1_{k'}\}))$ and, in this case, finite choice yields the $\Lambda^1(k)$ logic-system. Notice that the logic-system $\Lambda^1(k)$ is considered as a set-theoretic set. Then the logic-system algorithm $\mathcal{A}$ is applied to $(\Lambda^1(k), \{\mathrm{f}^1(0,0)\})$, where $\mathrm{f}^1(0,0)$ is the only hypothesis contained in the logic-system. This yields $\mathrm{f}^1(i,j) \in \mathcal{I}_1$ as a deduction from $\mathrm{f}^1(0,0)$. Conversely, if $\mathrm{f}^1(i,j) \in \mathcal{I}_1$, then there is an $\mathrm{X}^1_{k'} \in \mathrm{M}^1_{k'}$ and a logic-system $\Lambda(k) \in \mathcal{A}'(\mathcal{S}^1, \{\mathrm{X}^1_{k'}\})$ such that application of the logic-system algorithm $\mathcal{A}$ to $(\Lambda^1(k), \{\mathrm{f}^1(0,0)\})$ yields $\mathrm{f}^1(i,j)$ as a deduction from $\mathrm{f}^1(0,0)$.

The informal algorithm $\mathcal{A}$ is defined on any logic-system that contains an hypothesis and, in this paper, such a logic-system is $\Lambda^q(x)$ and application is on $(\Lambda^q(x), \mathrm{Y})$ where Y is an hypothesis contained in the logic-system and containing but one member. Due to the construction of the $\Lambda^q(x)$, this yields a partial sequence of members of $\mathcal{I}_q$. This sequence is denoted by $\mathcal{A}[(\Lambda^q, \mathrm{Y})]$. This sequence represents the steps in the deduction and satisfies the $\leq_{\mathcal{I}_x^q}$ order. Also, for this case, $\mathcal{A}((\Lambda^q(x), \mathrm{Y})) = \mathcal{I}_x^q \subset \mathcal{I}_q$. Significantly, for $n,\ k \in \mathbb{N},\ n \leq k, \mathcal{A}((\Lambda^1(n), \mathrm{Y})) \subset \mathcal{A}((\Lambda^1(k), \mathrm{Y}))$ and $\mathcal{A}[(\Lambda^1(k), \mathrm{Y})]|[1, n] = \mathcal{A}[(\Lambda^1(n), \mathrm{Y})]$.

In the usual way, all of the above informally defined objects are embedded relative to $\mathcal{E}$. When the informal set-theoretic expresses are considered as embedded into the standard superstructure, all of the bold font conventions defined in Herrmann (1979-1993) are observed. All other embedded symbols retain their math-italics form. Where script notation is used, an underline is used in place of the bold face font. All the following results are relative to our nonstandard model ${}^*\mathcal{M} = \langle\ {}^*\mathbf{Q}, \in, = \rangle$ or ${}^*\mathcal{M} = \langle\ {}^*\mathbb{R}, \in, = \rangle$ (Herrmann, (1979 - 1993)).

Theorem 2.1 *Consider primitive time interval* $1 = [0, b], b > 0$. *It can always be assumed that interval 1 is partitioned into two or more intervals* $[c_0, c_1), \ldots [c_{m-1}, c_m],\ c_m = b,\ m > 1,\ m \in \mathbf{Z}$. *Let* $\underline{\mathcal{I}}_1$ *be an instruction paradigm order isomorphic to the rational numbers* $R_1 \subset [0, b]$. *For any* $\lambda \in \mathbb{N}_\infty$, *there exists a unique hyperfinite* ${}^*\mathbf{\Lambda}^1(\lambda) \in {}^*\underline{\mathcal{L}}^1$ *and a* $\lambda' \in {}^*\mathbb{N}$ *such that the ultra-word-like* $X^1_{\lambda'} \in {}^*\mathbf{M}^1_{\lambda'}$ *and ultra-logic-system* ${}^*\mathbf{\Lambda}^1(\lambda) \in {}^*\underline{\mathcal{A}}'(({}^*\underline{\mathcal{S}}^1, \{X^1_{\lambda'}\}))$ *and* ${}^\sigma\underline{\mathcal{I}}_1 \subset {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0,0)\})) = \mathcal{I}^1_\lambda \subset {}^*\underline{\mathcal{I}}_1$. *Also the* ${}^*\underline{\mathcal{A}}[({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0,0)\})]$ *steps satisfy the $\leq_{\mathcal{I}^1_\lambda}$ order and* $({}^*\underline{\mathcal{I}}_1 - {}^\sigma\underline{\mathcal{I}}_1) \cap {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0,0)\})) =$ *an infinite set.*

Proof. This follows in the same manner as Theorem 4.1 in Herrmann (2006) by *-transfer of the appropriate first-order statements that precede this theorem statement. Also note that since for every $n \in \mathbb{N}'$, the $\Lambda(n)$ is finite, then, via the identification process, ${}^\sigma\mathbf{\Lambda}(n) = \mathbf{\Lambda}(n)$. It also follows that ${}^*\mathbf{\Lambda}(n) = \mathbf{\Lambda}(n)$ under the customary conventions. Since for any $n,\ k \in \mathbb{N}',\ n \leq k$, $\mathcal{A}((\mathbf{\Lambda}(n), \{\mathbf{f}^1(0,0)\})) \subset \mathcal{A}((\mathbf{\Lambda}(k), \{\mathbf{f}^1(0,0)\}))$, from the above and, via *-transfer, it follows that ${}^\sigma\underline{\mathcal{I}}_1 \subset {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(\lambda).\{{}^*\mathbf{f}^1(0,0)\})) = \mathcal{I}^1_\lambda \subset {}^*\underline{\mathcal{I}}_1$. From the definition of $\Lambda^1(n)$, these steps numbers are order isomorphic the set of rational numbers R_1. Hence, ${}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0,0)\}))$ is *order isomorphic to a hyperfinite subset of ${}^*\mathbf{Q}$. Since there are infinitely many $i < \lambda$ and $i \in \mathbb{N}_\infty$, there are infinitely many ${}^*\mathbf{f}(i,j) \in {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0,0)\})) \subset {}^*\underline{\mathcal{I}}_1$, where ${}^*\mathbf{f}(i,j) \in {}^*\underline{\mathcal{I}}_1 - {}^\sigma\underline{\mathcal{I}}_1$. These are interpreted as ultranatural events but in some cases may differ from physical events only in their primitive time identifications. This completes the proof. ▮

By considering the definition of $\mathcal{L}^1$, it follows that the given $1 < m \in \mathbb{N}$, ${}^*\mathbf{\Lambda}^1(\lambda)$ is precisely $\{{}^*\mathbf{f}^1(0,0)\} \cup \{\bigcup\{{}^*\mathbf{k}^1_i(\lambda) \mid 0 \leq i < m\}\} \cup \{\{{}^*\mathbf{f}^1(p-1, \lambda),\ {}^*\mathbf{f}^1(p, 0)\} \mid (0 < p \leq m) \wedge (p \in {}^*\mathbf{Z})\}$. Of significance is the fact that the steps in the *-deduction ${}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0,0)\}))$ preserve the order $\leq_{{}^*\underline{\mathcal{I}}_1}$. Notice that ${}^*\mathbf{\Lambda}^1(\lambda)$ is obtained by hyperfinite choice. Further, any ${}^*\mathbf{f}^1(i,j) \in \{{}^*\mathbf{f}^1(x,y) \mid (0 \leq x < m) \wedge (0 \leq y \leq \lambda) \wedge (x \in {}^*\mathbf{Z}) \wedge (y \in {}^*\mathbb{N})\} \cup \{{}^*\mathbf{f}^1(m,0)\}$ is a hyperfinite *-deduction from $\mathbf{f}^1(0,0) = {}^*\mathbf{f}^1(0,0)$. And, it also follows that the set of all such *deductions yields a hyperfinite set $\mathcal{I}^1_\lambda$ such that ${}^\sigma\underline{\mathcal{I}}_1 \subset \mathcal{I}^1_\lambda \subset {}^*\underline{\mathcal{I}}_1$.

3. Logic-System Generation for the Type-2 Interval

For the type-2 case $[0, +\infty)$, a denumerable instruction paradigm displays a refined form. For this case, $\mathcal{I}_2 = \{\mathrm{f}^2(i,j) \mid (0 \leq i) \wedge (i \in \mathbf{Z}) \wedge (j \in \mathbb{N})$. Using $\mathcal{I}_2$, consider the following logic-system.

Definition 3.1 Let $0 \leq i \in \mathbf{Z}$. For each $n \in \mathbb{N}$, let $\mathrm{k}_i^2(n) = \{\{\mathrm{f}^2(i,j), \mathrm{f}^2(i,j+1)\} \mid (0 \leq j \leq n-1) \wedge (j \in \mathbb{N})\}$. For $0 < m \in \mathbf{Z}$, let $\mathrm{K}^2(m,n) = \bigcup\{\mathrm{k}_i^2(n) \mid (0 \leq i < m) \wedge (i \in \mathbf{Z})\}$. Finally, let $\Lambda^2(m,n) = \{\mathrm{f}^2(0,0)\} \cup \mathrm{K}^2(m,n) \cup \{\{\mathrm{f}^2(p-1,n), \mathrm{f}^2(p,0)\} \mid (0 < p \leq m) \wedge (p \in \mathbf{Z})\} \cup \{\{\mathrm{f}^2(m,j), \mathrm{f}^2(m,j+1)\} \mid (0 \leq j < n) \wedge (j \in \mathbb{N})\}$, and $\mathcal{L}^2 = \{\Lambda^2(x,y) \mid (0 \leq x \in \mathbf{Z}) \wedge (y \in \mathbb{N})\}$. Notice that if $0 \leq i < k,\ i,\ k \in \mathbf{Z}$, then $\mathcal{A}((\Lambda^2(i,j), \{\mathrm{f}^2(0,0)\})) \subset \mathcal{A}((\Lambda^2(k,n), \{\mathrm{f}^2(0,0)\}))$ for any $j,\ n \in \mathbb{N}$. Also, each $\Lambda^2(m,n)$ is a finite set. (Notice that members in $\mathcal{L}^2$ can be characterized by a first-order sentence.)

Consider any $\Lambda^2(q,k)$. Then there exists an $q'k' \in \mathbb{N}'$ ($q'k'$ is a natural number in $\mathbb{N}'$) and the $q'k'$-set $\mathrm{X}^2_{q'k'} \in \mathrm{M}^2_{q'k'}$, such that $\Lambda^2(q,k) \in \mathcal{S}^2(\{\mathrm{X}^2_{q'k'}\})$ and, in this case, finite choice yields the $\Lambda^2(q,k)$ logic-system. Then the logic-system algorithm $\mathcal{A}$ applied to $(\Lambda^2(q,k), \{\mathrm{f}^2(0,0)\})$ yields $\mathrm{f}^2(q,k)$ as a deduction from $\mathrm{f}^2(0,0)$. Further, $\mathrm{f}^2(q,k) \in \mathcal{I}_2$. Conversely, if $\mathrm{f}^2(q,k) \in \mathcal{I}_2$, then there exists an $q'k' \in \mathbb{N}'$ and an $\mathrm{X}^2_{q'k'} \in \mathrm{M}^2_{q'k'}$ and a logic-system $\Lambda(q,k) \in \mathcal{A}'((\mathcal{S}^2, \{\mathrm{X}^2_{q'k'}\}))$ such that application of the logic-system algorithm $\mathcal{A}$ to $(\Lambda^2(q,k), \{\mathrm{f}^2(0,0)\})$ yields a deduction of $\mathrm{f}^2(q,k)$ from $\mathrm{f}^2(0,0)$.

Theorem 3.1 *Consider primitive time interval* $2 = [0,+\infty)$. *It can always be assumed that interval 2 is partitioned into intervals* $[c_0,c_1), \ldots [c_{m-1},c_m),\ m > 1,\ m \in \mathbf{Z}$. *Let* $\mathbf{d}_2$ *be an instruction paradigm order isomorphic to the rational numbers* $R_2 \subset [0,+\infty)$. *For any* $\lambda \in \mathbb{N}_\infty$ *and* $\nu \in {}^*\mathbf{Z} - \mathbf{Z},\ \nu > 0$, *there exists a unique hyperfinite* ${}^*\mathbf{\Lambda}^2(\nu,\lambda) \in {}^*\underline{\mathcal{L}}^2$ *and* $\nu', \lambda' \in {}^*\mathbb{N}$ *such that the ultra-word-like* $X^2_{\nu'\lambda'} \in {}^*\mathbf{M}^2_{\nu'\lambda'}$ *and ultra-logic-system* ${}^*\mathbf{\Lambda}^2(\nu,\lambda) \in {}^*\underline{\mathcal{A}}'(({}^*\mathcal{S}^2, \{X^2_{\nu'\lambda'}\}))$ *and* ${}^\sigma\underline{\mathcal{I}}_2 \subset {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^2(\nu,\lambda), \{{}^*\mathbf{f}^2(0,0)\})) = \mathcal{I}^2_{\nu\lambda} \subset {}^*\underline{\mathcal{I}}_2$. *Also the* ${}^*\underline{\mathcal{A}}[({}^*\mathbf{\Lambda}^2(\nu,\lambda), \{{}^*\mathbf{f}^2(0,0)\}))]$ **steps satisfy the* $\leq_{\mathcal{I}^2_{\nu\lambda}}$ *order and* $({}^*\underline{\mathcal{I}}_2 - {}^\sigma\underline{\mathcal{I}}_2) \cap {}^*\underline{\mathcal{A}}({}^*\mathbf{\Lambda}^2(\nu,\lambda), \{{}^*\mathbf{f}^2(0,0)\})) =$ *an infinite set.*

Proof. As in Theorem 2.1, the proof follows by *-transfer of the appropriate formally presented material that appears above in this section 3.

By considering the definition of $\mathcal{L}^2$, it follows that the ${}^*\mathbf{\Lambda}^2(\nu,\lambda)$ is precisely $\{{}^*\mathbf{f}^2(0,0)\} \cup \{\bigcup\{{}^*\mathbf{k}_i^2(\lambda) \mid 0 \leq i < \nu\}\} \cup \{(\{{}^*\mathbf{f}^2(p-1,\lambda), {}^*\mathbf{f}^2(p,0)\} \mid (0 < p \leq \nu) \wedge (p \in {}^*\mathbf{Z})\} \cup \{\{{}^*\mathbf{f}^2(\nu,j), {}^*\mathbf{f}^2(\nu,j+1)\} \mid (0 \leq j < \lambda) \wedge (j \in {}^*\mathbb{N})\}$. Of significance is the fact that the steps in the *-deduction ${}^*\underline{\mathcal{A}}[{}^*\mathbf{\Lambda}^2(\nu,\lambda), \{{}^*\mathbf{f}^2(0,0)\}))]$ preserve the order $\leq_{{}^*\underline{\mathcal{I}}_2}$. Notice that ${}^*\mathbf{\Lambda}^2(\nu,\lambda)$ is obtained by hyperfinite choice. Further, any ${}^*\mathbf{f}^2(i,j) \in \{{}^*\mathbf{f}^2(x,y) \mid (0 \leq x \leq \nu) \wedge (0 \leq y \leq \lambda) \wedge (x \in {}^*\mathbf{Z}) \wedge (y \in {}^*\mathbb{N})\}$ is a hyperfinite *-deduction from $\mathbf{f}^2(0,0)$. And, it also follows that the set of all such *deductions yield a hyperfinite set $\mathcal{I}^2_{\nu\lambda}$ such that ${}^\sigma\underline{\mathcal{I}}_2 \subset \mathcal{I}^2_{\nu\lambda} \subset {}^*\underline{\mathcal{I}}_2$.

4. Logic-System Generation for the Type-3 Interval

For the type-3 case $(-\infty,0]$, a denumerable instruction paradigm displays a refined form. For this case, $\mathcal{I}_3 = \{\mathrm{f}^3(i,j) \mid (i \leq 0) \wedge (i \in \mathbf{Z}) \wedge (j \in \mathbb{N})$. Using $\mathcal{I}_3$, consider the following logic-system.

Definition 4.1 Let $i \in \mathbf{Z},\ i \leq 0$. For each $n \in \mathbb{N}$, let $\mathrm{k}_i^3(n) = \{\{\mathrm{f}^2(i,j), \mathrm{f}^1(i,j+1)\} \mid (0 \leq j \leq n-1) \wedge (j \in \mathbb{N})\}$. For $m \in \mathbf{Z}\, m < 0$, let $\mathrm{K}^3(m,n) = \bigcup\{\mathrm{k}_i^3(n) \mid (m \leq i < 0) \wedge (i \in \mathbf{Z})\}$. Finally, let $\Lambda^3(m,n) = \{\mathrm{f}^3(m,0)\} \cup \mathrm{K}^3(m,n) \cup \{\{\mathrm{f}^3(p-1,n), \mathrm{f}^3(p,0)\} \mid (m < p \leq 0) \wedge (p \in \mathbf{Z})\}$, and $\mathcal{L}^3 = \{\Lambda^2(x,y) \mid (0 \leq x \in \mathbf{Z}) \wedge (y \in \mathbb{N})\}$. Notice that if $i < k \leq 0,\ i,\ k \in \mathbf{Z}$, then $\mathcal{A}((\Lambda^3(i,j)), \{\mathrm{f}^3(m,0)\})) \subset \mathcal{A}((\Lambda^3(k,n), \{\mathrm{F}^3(m,0)\}))$ for any $j,\ n \in \mathbb{N}$. Also, each $\Lambda^3(m,n)$ is a finite set. (Notice that members in $\mathcal{L}^3$ can be characterized by a first-order sentence.)

Consider any $\Lambda^3(q,k)$. Then there exists an $q'k' \in \mathbb{N}$ and $\mathrm{X}^3_{q'k'} \in \mathrm{M}^3_{q'k'}$, such that $\Lambda^3(q,k) \in \mathcal{A}'((\mathcal{S}^3, \{\mathrm{X}^3_{q'k'}\}))$ and, in this case, finite choice yields the $\Lambda^3(q,k)$ logic-system. Then the logic-system algorithm $\mathcal{A}$ applied to $(\Lambda^3(q,k), \{\mathrm{f}^3(q,0)\}))$ yields $\mathrm{f}^3(q,k)$ as a deduction from $\mathrm{f}^3(q,0)$.

Further, $f^3(q,k) \in \mathcal{I}_3$. Conversely, if $f^3(q,k) \in \mathcal{I}_3$, then there is an $X^3_{q'k'} \in \mathcal{M}^3_{q'k'}$ and a logic-system $\Lambda(q,k) \in S^3(\{X^3_{q'k'}\})$ such that application of the logic-system algorithm $\mathcal{A}$ to $(\Lambda^3(q,k), \{{}^*\mathbf{f}^3(q,0)\}))$ yields $f^3(q,k)$ as a deduction from $f^3(q,0)$.

Theorem 4.1 *Consider primitive time interval* $3 = (-\infty, 0]$. *It can always be assumed that interval 3 is partitioned into intervals* $\ldots,[c_{-2},c_{-1}),[c_{-1},c_0]$. *Let* $\mathbf{d}_3$ *be an instruction paradigm order isomorphic to the rational numbers* $R_3 \subset (-\infty,0]$. *For any* $\lambda \in \mathbb{N}_\infty,\ \mu \in {}^*\mathbf{Z}-\mathbf{Z},\ \mu < 0$, *there exists a unique hyperfinite* ${}^*\mathbf{\Lambda}^3(\mu,\lambda) \in {}^*\underline{\mathcal{L}}^3$ *and* $\mu',\lambda' \in {}^*\mathbb{N}$ *such that the ultra-word-like* $X^3_{\mu'\lambda'} \in {}^*\mathbf{M}^3_{\mu'\lambda'}$ *and ultra-logic-system* ${}^*\mathbf{\Lambda}^3(\mu,\lambda) \in {}^*\underline{\mathcal{A}}'(({}^*\mathcal{S}^3,\{X^3_{\mu'\lambda'}\}))$ *and* ${}^\sigma\underline{\mathcal{I}}_3 \subset {}^*\underline{\mathcal{A}}({}^*\mathbf{\Lambda}^3(\mu,\lambda),\{{}^*\mathbf{f}^3(\mu,0)\})) = \mathcal{I}^3_{\mu\lambda} \subset {}^*\underline{\mathcal{I}}_3$. *Also the* ${}^*\underline{\mathcal{A}}[({}^*\Lambda^3(\mu,\lambda),\{{}^*\mathbf{f}^3(\mu,0)\}))]$ **steps satisfy the* $\leq_{\mathcal{I}^3_{\mu\lambda}}$ *order and* $({}^*\underline{\mathcal{I}}_3 - {}^{\mathcal{I}}{}_3) \cap {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^3(\mu,\lambda)),\{{}^*\mathbf{f}^3(\mu,0)\})) =$ *an infinite set.*

Proof. As in Theorem 3.1, the proof follows by *-transfer of the appropriate formally presented material that appears above in this section 3.

By considering the definition of $\mathcal{L}^3$, it follows that the ${}^*\mathbf{\Lambda}^3(\mu,\lambda)$ is precisely $\{{}^*\mathbf{f}^3(\mu,0)\} \cup \{\bigcup\{{}^*\mathbf{k}^3_i(\lambda) \mid \mu \leq i < 0\}\} \cup \{\{{}^*\mathbf{f}^3(p-1,\lambda),\ {}^*\mathbf{f}^3(p,0)\} \mid (\mu < p \leq 0) \wedge (p \in {}^*\mathbf{Z})\}$. Of significance is the fact that the steps in the *-deduction ${}^*\underline{\mathcal{A}}[({}^*\mathbf{\Lambda}^3(\mu,\lambda),\{{}^*\mathbf{f}^3(\mu,0)\}]$ preserve the order $\leq_{{}^*\underline{\mathcal{I}}_3}$. Notice that ${}^*\mathbf{\Lambda}^3(\mu,\lambda)$ is obtained by hyperfinite choice. Further, any ${}^*\mathbf{f}^3(i,j) \in \{{}^*\mathbf{f}^3(x,y) \mid (\mu \leq x < 0) \wedge (0 \leq y \leq \lambda)\} \cup \{\mathbf{f}^3(0,0\}$ is a hyperfinite *-deduction from $\mathbf{f}^3(\mu,0)$. And, it also follows that the set of all such *deductions is a hyperfinite set $\mathcal{I}^3_{\nu\lambda}$ such that ${}^\sigma\underline{\mathcal{I}}_3 \subset \mathcal{I}^3_{\nu\lambda} \subset {}^*\underline{\mathcal{I}}_3$.

5. Logic-System Generation for the Type-4 Interval

Theorem 5.1 *Consider primitive time interval* $4 = (-\infty,+\infty)$. *It can always be assumed that interval 4 is partitioned into intervals* $\ldots,[c_{-2},c_{-1}),[c_{-1},c_0),\ldots$. *Let* $\mathbf{d}_4$ *be a instruction paradigm order isomorphic to the rational numbers* $R_4 \subset (-\infty,+\infty)$. *For any* $\lambda \in \mathbb{N}_\infty,\ \nu,\gamma \in {}^*\mathbf{Z}-\mathbf{Z}$, *such that* $\nu \leq 0,\ \gamma \geq 0$, *there exists a unique hyperfinite* ${}^*\mathbf{\Lambda}^4(\nu,\gamma,\lambda) \in {}^*\underline{\mathcal{L}}^4$ *and* $\nu',\gamma',\lambda' \in {}^*\mathbb{N}$ *such that the ultra-word-like* $X^4_{\nu'\gamma'\lambda'} \in {}^*\mathbf{M}^4_{\nu'\gamma'\lambda'}$ *and ultra-logic-system* ${}^*\mathbf{\Lambda}^4(\nu,\gamma,\lambda) \in {}^*\underline{\mathcal{A}}'(({}^*\mathcal{S}^4,\{X^4_{\nu'\gamma'\lambda'}\}))$ *and* ${}^\sigma\underline{\mathcal{I}}_4 \subset {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^4(\nu,\gamma,\lambda),\{{}^*\mathbf{f}^4(\nu,0)\})) = \mathcal{I}^4_{\nu\gamma\lambda} \subset {}^*\underline{\mathcal{I}}_4$. *Also the* ${}^*\underline{\mathcal{A}}[({}^*\mathbf{\Lambda}^4(\nu,\gamma,\lambda),\{{}^*\mathbf{f}^4(\nu,0)\})]$ **steps satisfy the* $\leq_{\mathcal{I}^4_{\nu\gamma\lambda}}$ *order and* $({}^*\underline{\mathcal{I}}_4 - {}^\sigma\underline{\mathcal{I}}_4) \cap {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^4(\nu,\gamma,\lambda),\{{}^*\mathbf{f}^4(\nu,0)\})) =$ *an infinite set.*

By considering the definition of $\mathcal{L}^4$, it follows that the ${}^*\mathbf{\Lambda}^4(\nu,\gamma,\lambda)$ is precisely $\{{}^*\mathbf{f}^4(\nu,0)\} \cup \{\bigcup\{{}^*\mathbf{k}^4_i(\lambda) \mid (\nu \leq i < \gamma) \wedge (i \in {}^*\mathbf{Z})\}\} \cup \{\{{}^*\mathbf{f}^4(p-1,\lambda),\ {}^*\mathbf{f}^4(p,0)\} \mid (\nu < p \leq \gamma) \wedge (p \in {}^*\mathbf{Z})\} \cup \{\{{}^*\mathbf{f}^4(\gamma,j),\ {}^*\mathbf{f}^4(\gamma,j+1)\} \mid (0 \leq j < \lambda) \wedge (j \in {}^*\mathbb{N})\}$. Of significance is the fact that the steps in the *-deduction ${}^*\underline{\mathcal{A}}[({}^*\mathbf{\Lambda}^4(\nu,\gamma,\lambda),\{{}^*\mathbf{f}^4(\nu,0)\})]$ preserve the order $\leq_{{}^*\underline{\mathcal{I}}_4}$. Notice that ${}^*\mathbf{\Lambda}^4(\nu,\gamma,\lambda)$ is obtained by hyperfinite choice. Further, any ${}^*\mathbf{f}^4(i,j) \in \{{}^*\mathbf{f}^4(x,y) \mid (\nu \leq x \leq \gamma) \wedge (0 \leq y \leq \lambda)\}$ is a hyperfinite *-deduction from $\mathbf{f}^4(\nu,0)$. And, it also follows that the set of all such *deductions is a hyperfinite set $\mathcal{I}^4_{\nu\gamma\lambda}$ such that ${}^\sigma\underline{\mathcal{I}}_4 \subset \mathcal{I}^4_{\nu\gamma\lambda} \subset {}^*\underline{\mathcal{I}}_4$.

6. Necessary refinements.

For the GGU-model, a universe is a nonempty collection of empty-systems, physical-systems, physical-like systems or other-systems. In general, an infinite hyperfinite set X of internal sets, disjoint from $\bigcup\{{}^*\mathcal{F}^i(\mathcal{C}) \mid (1 \leq i \leq n) \wedge (i \in \mathbb{N})\},\ n > 0$, is adjoined when info-fields are employed. Let $\mathcal{K} = \mathcal{P}(\bigcup\{{}^*\mathcal{F}^i(\mathcal{C}) \mid (1 \leq i \leq n) \wedge (i \in \mathbb{N})\} \cup X)$.

It is now necessary that a more refined definition for each $\mathbf{f}^q(i,j)$, which yields each ${}^*\mathbf{f}^q(i.j)$, be given. The notion of the "non-operative" instruction is used. Using the alphabet symbol $\mathrm{yX} \in \mathcal{W}'$, consider the meaningless "word" yX. If this word is considered an instruction, then it has no operative content and **yX** yields neither properton combinations of any form nor any entities that require the adjoined set X. Its application yields an empty-system. This word is introduced in order to simplify the following refinement.

Although thus far ${}^*\mathbf{f}^q(i,j)$ has been considered as an *instruction-set and any further refinements as to how it is constructed were unnecessary, this is no longer the case. None of the previous results are altered by this refinement. All members of ${}^*\mathcal{E}'$ being considered in this section are *instructions. Let $\mathbf{Z}_q,\ q = 1,2,3,4$, be as employed to define the $\mathcal{I}_q$. Consider, as with q and defined in the same manner, $\mathbf{Z}_r \subset \mathbf{Z}$, where $r = 1,2,3,4$. Notationally, let $\mathbf{T} = \mathbf{Z}_r \times \mathbb{N}$. Both $\mathbf{Z}_q \times \mathbb{N}$ and $\mathbf{T}$ carry the simple lexicographic order $\preceq$ (Herrmann, 2006).

In what follows, given a particular q, then r is fixed for all universe-wide frozen-frames. There exists a function $\mathrm{g}^{(q,r)}\colon \mathbf{Z}_q \times \mathbb{N} \to (\mathcal{F}'(\mathcal{W}'))^{\mathbf{T}}$ with the following properties. For $(i,j) \in \mathbf{Z}_r \times \mathbb{N}$, $\mathrm{g}^{(q,r)}(i,j) = \mathrm{v}$. Then for each $(k,s) \in \mathbf{Z}_r \times \mathbb{N}$, $\mathrm{v}(k,s)$ is an appropriate instruction-set. Notationally, the instruction-set is $(\mathrm{g}^{(q,r)}(i,j))(k,s) = \mathrm{v}(k,s) = \mathrm{g}^{(q,r)}(i,j;k,s)$.

Using this notation, for a fixed (i,j), the $\mathrm{g}^{(q,r)}(i,j;k,s)$ has the same type of primitive time ordering (lexicographic) as does the universe-wide frozen-frames. For each $(i,j,k) \in \mathbf{Z}_q \times \mathbb{N} \times \mathbf{Z}_r$, let $\mathrm{h}^{(q,r)}(i,j;k) = \bigcup\{\mathrm{g}^{(q,r)}(i,j;k,s) \mid s \in \mathbb{N}\}$ and, for each $(i,j) \in \mathbf{Z}_q \times \mathbb{N}$, let the instruction-set $\mathrm{f}^q(i,j) = \bigcup\{\mathrm{h}^{(q,r)}(i,j;k) \mid k \in \mathbf{Z}_r\}$. By embedding and *-transfer, ${}^*(\mathbf{g}^{(q,r)}(i,j))(k,s) = {}^*\mathbf{v}(k,s) = {}^*\mathbf{g}^{(q,r)}(i,j;k,s)$. Then, for each $(i,j,k) \in {}^*\mathbf{Z}_q \times {}^*\mathbb{N} \times {}^*\mathbf{Z}_r$, ${}^*\mathbf{h}^{(q,r)}(i,j;k) = \bigcup\{{}^*\mathbf{g}^{(q,r)}(i,j;k,s) \mid s \in {}^*\mathbb{N}\}$ and the *instruction-set ${}^*\mathbf{f}^{(q,r)}(i,j) = \bigcup\{{}^*\mathbf{h}^{(q,r)}(i,j,k) \mid k \in {}^*\mathbf{Z}_r\}$. The ${}^*\preceq$ *ordering is also satisfied.

With respect to the original method used to obtained words in $\mathcal{W}'$, note that from the definitions and when considered as restrictions, for fixed (i,j,k), each $\mathrm{g}^{(q,r)}(i,j;k,r)$ is an instruction-set, each $\mathrm{h}^{(q,r)}(i,j;k)$ is an instruction-set (of instruction-sets) as is each $\mathrm{f}^{(q,r)}(i,j)$. As previously mentioned, for fixed (i,j), (k,s) there is a single word $\mathrm{W}^{(q,r)}_{(i,j;k,s)}$ in $\mathcal{W}'$ that corresponds to $\mathrm{g}^{(q,r)}(i,j;k,s)$. Then for each k there is a word $\mathrm{W}^{(q,r)}_{(i,j;k)} \in \mathcal{W}'$ corresponding to $\mathrm{h}^{(q,r)}(i,j;k)$ that is formed by ordering intuitively from right-to-left, via the word theory join operator, the individual symbol-strings, diagrams, images or sensory information determined by each word $\mathrm{g}^{(q,r)}(i,j;k,s)$ as s varies via the linear portion of the lexicographic order. This gives a word in $\mathcal{W}'$ composed of the individual words. This word yields in a written or thought-order (left-to-right or right-to-left depending upon the language) set of instructions for each of the finite k systems. However, the speaking-order is fixed.

In like manner for fixed (i,j), there is a member of $\mathcal{W}'$ that yields, in written or thought word-order form, via the lexicographic order, a single word $\mathrm{W}^{(q,r)}_{(i,j)} \in \mathcal{W}'$. Then there is a single word $\mathrm{W}^{(q,r)}(n)$ that yields in written or thought word-order form, again via the lexicographic order, a word that corresponds to a complete development for a finitely generated universe. But, notice that the speaking-order or corresponding thought-.order is fixed. These results are immediately extended to the hyperfinite cases. Then the finite cases for the $\mathrm{W}^{(q,r)}(n)$ immediately yield the existence of an ultraword $W^{(q,r)}(\lambda)$ with the same first-order properties as $\mathrm{W}^{(q,r)}(n)$. This can be considered as a higher-form of an hyper-ordered "spoken" or "thought" word. This has theological applications.

For each $(i,j) \in {}^*\mathbf{Z}_q \times {}^*\mathbb{N}$ and $k \in {}^*\mathbf{Z}_r$, ${}^*\mathbf{h}^{(q,r)}(i,j;k)$ yields an empty-system, physical-system, physical-like system or an other-system. (Note that physical-like properties include physical-like behavior relative to non-physical entities, where the entities have no other known properties.) Of course, each ${}^*\mathbf{f}^{(q,r)}(i,j)$ is an *instruction-set as is each ${}^*\mathbf{h}^{(q,r)}(i,j;k)$. By application of the word **yX** and the GGU-model construction of each universe-wide frozen frame, there are only finitely or hyperfinitely many physical or physical-like systems. Further more, various physical-like systems can be physical-systems in that only physical events exist. Which physical-like systems are but physical depends upon choice since the GGU-model is not dependent upon what science classifies as "physical." Such a choice is dependent upon a chosen interpretation. The same holds for what are considered as empty-systems or other-systems.

For each $(i,j) \in {}^*\mathbf{Z}_q \times {}^*\mathbb{N}$ that yields a universe-wise frozen-frame, ${}^*\mathbf{g}^{(q,r)}(i,j;k,s)$ produces an info-field in the same manner as an entire universe is considered associated with a *developmental paradigm. These info-fields can also be considered in system form via the ${}^*\mathbf{h}^{q,r)}(i,j;k)$ *instruction-sets. If empty-systems are used, only a type-r = 4 ultra-logic-system need be considered. In this case, this yields for each such (i,j), and fixed $\{\mu,\gamma,\lambda\} \subset {}^*\mathbb{N}$, a generating ultra-logic-system is

$$
{}^*\mathbf{F}^{(q,4)}(i,j) = \{ {}^*\mathbf{g}^{(q,4)}(i,j;\nu,0)\} \cup \{\bigcup\{ {}^*\mathbf{k}_x^{(q,4)}(\lambda) \mid (\nu \le x < \gamma) \wedge (x \in {}^*\mathbf{Z})\}\}\cup
$$

$$
\{\{ {}^*\mathbf{g}^{(q,4)}(i,j;k-1,\lambda),\ {}^*\mathbf{g}^{(q,4)}(i,j;k,0)\} \mid (\nu < k \le \gamma) \wedge (k \in {}^*\mathbf{Z}_4)\}\cup
$$

$$
\{\{ {}^*\mathbf{g}^{(q,4)}(i,j;\gamma,s),\ {}^*\mathbf{g}^{(q,4)}(i,j;\gamma,s+1)\} \mid (0 \le s < \lambda) \wedge (s \in {}^*\mathbb{N})\}.
$$

For this refined approach, the algorithm $\mathcal{A}$ has an extended definition. It is applied to these special logic-systems where the members are themselves logic-systems. This is how modified $\mathcal{A}'$ is applied. As each logic-system is obtained in the indicated ordered manner, $\mathcal{A}$ is applied to it. This is what would occur in the standard definition for application of $\mathcal{A}$ to a collection of logic-systems in n-tuple form except that the logic-systems are obtained deductively in a specific order and then as each is deduced the deduction is completed for the deduced logic-system. This is a rather natural way one would preceed. Under this extended definition for $\mathcal{A}$ all of the previous theorems in sections 2, 3, 4, 5 hold where F [resp. $\mathbf{F}$, ${}^*\mathbf{F}$] replaces f [resp. $\mathbf{f}$, ${}^*\mathbf{f}$].

Let $D_i \subset \mathbb{R}$ be a countable set of non-zero numerical or coded values for a particular ultra-properton coordinate $i \in K$. Let $D = \bigcup\{D_i \mid i \in K\}$. The set $G(0)$ is the set of all finite numbers. For each $r \in D$, let $\Lambda_p = \{x \mid (x \in G(0)) \wedge (\mathtt{st}(x/10^\omega) = p)\}$. The sets Λ_p are disjoint and by choice consider distinct $\lambda_p \in \Lambda_p$ for each $p \in D$. For any $p \in D$, a set P_p of ultra-propertons is λ_p-finite, if P_p is hyperfinite and there exists a bijection from $[1,\lambda_p]$ onto P_p. From the definition of the set of all ultra-propertons, for each $i',j' \in [1,K]$, and each $p,y \in D$, there exist a λ_p-finite $x(p,i') \in {}^*\mathcal{F}(\mathcal{C})$ and a λ_y-finite $x(y,j') \in {}^*\mathcal{F}(\mathcal{C})$ such that if $i \neq j$, then $x(p,i') \cap x(p,j') = \emptyset$. Of course, there are distinct λ_y-finite sets that yield physical or physical-like events.

> All of the previous results hold for the case of a corresponding developmental paradigm by substituting corresponding developmental paradigm $\mathrm{f}^{(q,r)}$, $\mathrm{g}^{(q,r)}(i,j;k,s)$, and the like for an instruction paradigm notation.

There is a set-function

$$
G_z^{(q,r)}\colon {}^*\mathcal{P}(\{ {}^*\mathbf{g}^{(q,r)}(i,j;k,s) \mid (\alpha \le i \le \beta) \wedge (i \in {}^*\mathbf{Z}_q) \wedge (j \in {}^*\mathbb{N})\wedge
$$

$$
(k \in {}^*\mathbf{Z}_r) \wedge (s \in {}^*\mathbb{N})\}) \to \Pi^+
$$

where α,β depend upon q. For this approach, one considers ${}^*\mathcal{P}(\{ {}^*\mathbf{g}^{(q,r)}(i,j;k,s) \mid (\alpha \le i \le \beta)\wedge(i \in {}^*\mathbf{Z}_q) \wedge (j \in {}^*\mathbb{N}) \wedge (k \in {}^*\mathbf{Z}_r) \wedge (s \in {}^*\mathbb{N})\}) \subset {}^*\mathcal{P}({}^*\mathcal{E}')$. Of course, an image can be the empty set.

The process of applying the standard part operator does not erase the original coordinates of the constructed objects. The process only yields the physical-world objects. Such alterations of the fundamental subatomic or field entities is modeled by vector space subtraction and this would be followed by independent *-finite subtraction. This represents a "breaking apart" of the fundamental entity into its original ultra-properton constituents. The original identifiers are restored. An altered constituent is then obtained by repeating the construction process. This can be used to eliminate the virtual particle or process concept within reality models. The models that need such concepts to predict behavior are now just that "models" for predicted behavior and would not require the virtual "stuff" to exist in reality.

Of course, for each (i,j), there is the important $G_z^{(q,r)}[\{\ {}^*\mathbf{h}^{(q,r)}(i,j;k)) \mid k \in \mathbf{Z}_r\}]$ that yields empty-systems, physical-systems, physical-like systems or other-systems. The set $G_z^{(q,r)}[\{\ {}^*\mathbf{h}^{(q,r)}(i,j;k) \mid k \in \mathbf{Z}_r\}]$ is an info-field. The $G_z^{(q,r)}$ represents a substratum medium and processes that yield this action.

Each member of the domain of $G_z^{(q,r)}$ can be replaced by a single member of ${}^*\mathcal{E}'$ since any nonempty finite subset set of $\mathcal{W}'$ can be replaced with a string of symbols connected by an & or a similar "conjuction" symbol. This new form, in this interpretation, has the same meaning as the set of finitely many members of $\mathcal{W}'$. This yields a single member of $\mathcal{E}'$ that can replace the finite instruction-set. Using strings of mathematical symbols that are members of ${}^*\mathcal{E}'$ the same can be done for each member of the range of $G_z^{(q,r)}$. This yields a new binary relation $\mathcal{G}_z^{(q,r)} \subset {}^*\mathcal{E}' \times {}^*\mathcal{E}'$. (Or by application of ${}^*w^{-1}$, ${}^*\mathcal{W}' \times {}^*\mathcal{W}'$

The binary relation $\mathcal{G}_z^{(q,r)}$ can be interpreted as a logic-system and as such the entire non-physical process can be interpreted as representing an intelligent design. Further, since the domain elements are so designed, then by implication so are the info-fields.

7. The Complete GGU-model Scheme

We assume that the previous results, not so generalized, have been generalized in order to accommodate the above necessary refinement . For the $(\mathtt{St}G_z^{(q,r)})$ and the $'\mathtt{St}$ defined in Herrmann (2006), the following scheme is not in composition notational form due to one application of choice and the step-by-step application of $(\mathtt{St}G_z^{(q,r)})$. It represents an ordered application of the GGU-model operators. For $q = 1,2,3,4$, the a,b,c take the appropriate value for a specific q.

$$(\mathtt{St}G_z^{(q,4)})(\ {}^*\underline{\mathcal{A}}((\ {}^*\mathbf{\Lambda}^{(q,4)}(a), \{\ {}^*\mathbf{F}^{(q,r)}(b,c)\})))(\ {}^*\underline{\mathcal{A}}'((\ {}^*\mathcal{S}^{(q,r)}, \{X_{a'}^{(q,r)}\}))).$$

The $\mathbf{\Lambda}^{(q,4)}(a)$ is obtained, as previously, except that ${}^*\mathbf{f}^q(i,j)$ is replaced with ${}^*\mathbf{F}^{(q,4)}(i,j)$. The operators $\underline{\mathcal{A}}$ and $\underline{\mathcal{A}}'$ have characterizing first-order statements. These statements need not capture all of of the intuitive statements that describe the algorithms. The results of application of $\underline{\mathcal{A}}'$ as formalized can show major aspects of the algorithm's selection process. For example, in terms of the corresponding (q,r) definitions,

$$\forall x\forall y\forall z\forall w((w \in \underline{\mathcal{M}}^{(q,r)} \land (y \in \mathcal{F}(\underline{\mathcal{L}}^{(q,4)})) \land (y \in w) \land (x \in \underline{\mathcal{A}}'((\underline{\mathcal{S}}^{(q,r)}, y)) \to$$

$$\exists p((p \in \underline{\mathcal{S}}^{(q,r)}) \land (y \in p) \land (x \in p) \land (y \neq x)).$$

For the instruction-information behavior (1 - 5), the function $G_z^{(q,r)}$ represents non-physical (2) and (3) and yields (4) the non-physical info-fields. For a particular universe and for each corresponding λ_p-finite $x(p,i')$ there is a set of *instructions $I(p,i')$ such that $G_z^{(q,r)}((I(p,i')) = x(p,i')$. These collections are considered as "bound." This binding is represented by application of λ_p-finite independent coordinate addition. The *instruction-sets that yield physical-systems contain various standard statements and these *instruction-sets also contain statements that refer to the non-physical.

The realization operator $\mathtt{st}(\cdot)$ is only applied to the λ_p-finite collections of ultra-propertons and, when applied, an entity represented by an intermediate properton is produced. From the modeling viewpoint, processes within the substratum are non-physical. Thus in summary, relative to the GGU-model and for comprehension, steps (2) and (3) are consider as statements relative to a non-physical medium and a non-physical mode of transmission, respectively. For each universe-wide frozen-frame, step (4) corresponds to a non-physical info-field. Step (5) is the application of the $\mathtt{st}(\cdot)$ operator and yields that physical products of the entire process. In order to be operative, the above displayed *instruction as well as similar ones have non-physical components.

The latest refinements and alterations, if any, to the GGU-model can be found in the Herrmann (2014, 2013) references.

8. The Participator Universe.

(The following is not presented in the generalized refined form.) For the GGU-model, one of the most difficult requirements is to include the concept of the "participator" universe. As stated at the May 1974 Oxford Symposium in Quantum Gravity, Patton and Wheeler describe how existence of human beings alter the universe to various degrees. "To that degree the future of the universe is changed. We change it. We have to cross out that old term 'observed' and replace it with the new term 'participator.' In some strange sense the quantum principle tells us that we are dealing with a participator universe." (Patton and Wheeler (1975, p. 562).) This aspect of the GGU-model is only descriptively displayed in section 4.8 in Herrmann (2002). It is now possible to obtain formally the collection of pre-designed universes that satisfies this participator requirement.

The previous notation is modified for finitely many (> 0) instruction paradigms as previously denoted by $\mathcal{I}_q$. From the construction of each instruction paradigm using $\mathcal{W}'$, it follows that there is, at least, a sequence of possible alterations. An instruction paradigm is a nonempty subset of the subsets $\mathcal{W}'$. Hence, the collection of all such instruction paradigms is a member of $\mathcal{P}(\mathcal{P}(\mathcal{W}'))$. There can be infinitely many basic universes. These are universes prior to participator alterations. For each of these, there is an collection of ultra-word-like objects of the appropriate type. What follows next is for an arbitrary member of this collection of ultra-world-like objects and, hence, an arbitrary basic universe.

So as to include the type of universe being considered, let $q\colon \mathbb{N}' \times [1,4] \to \mathcal{P}(\mathcal{P}(\mathcal{W}'))$. Then for a specific $p \in [1,4]$ an expression $\{x \mid (x = \mathrm{q}(n,p)) \wedge (n \in \mathbb{N}')\} = \mathcal{I}_p$ represents this denumerable set of instruction paradigms for type-p universes. (If $n' \neq m$, then $q(n',p) \neq q(m,p)$.) Let $\{x \mid \exists p(p \in [1,4] \wedge (x = \mathcal{I}_p)\} = \mathcal{I}$. Then, a specific $\mathrm{f}^1(0,0)$ is further identified relative to the sequences. As an example, $\mathrm{f}^{\mathrm{q}(3,1)}(0,0) \subset \mathcal{W}'$ represents a specific $n = 3$ type-1 instruction-set. Thus, as embedded into the formal structure this last expression reads $\mathbf{f}^{\mathbf{q}(3,1)}(0,0) \subset \mathcal{E}'$.

An original alteration can be miniscule and made in one or more of the necessary parameters that are satisfied by a specific cosmology. This can be done in such a way that only miniscule alterations in physical-system satisfy the alterations. On the other hand, a highly altered cosmology can also occur. An alteration is local prior to it being propagated during a universe's development. Although not specifically included, and indeed the definition would need to be altered slightly, other sequences q' can be used where various values can be empty or repeated. Also, for a universe with infinitely many local alterations at the same moment, one can include the obvious change in the q sequence, where the sequence q is a sequence of type-4 except that each image is a "universe."

For the GGU-model, the various members of $\mathcal{I}_p$ satisfy the "participator" requirements, when participators exist, for each of the known suggested cosmologies. When embedded into the formal structure, properties of q can be easily characterized, using various forms, in a first order language. For example,

$$\forall x \forall p((x \in \mathbb{N}') \wedge (p \in [1,4]) \to \exists y \exists z((z \in \underline{\mathcal{I}}) \wedge (y \in z) \wedge (\mathbf{q}(x,p) = y)).$$

Consider a specific $p' \in [1,4]$. Then

$$\forall y((y \in \underline{\mathcal{I}}_{p'})) \to \exists x((x \in \mathbb{N}') \wedge (\mathbf{q}(x,p') = y)).$$

In *-transfer form, these two sentences read

$$\forall x \forall p((x \in {}^*\mathbb{N}') \wedge (p \in [1,4]) \to \exists y \exists z((z \in {}^*\underline{\mathcal{I}}) \wedge (y \in z) \wedge ({}^*\mathbf{q}(x,p) = y)).$$

$$\forall y((y \in {}^*\underline{\mathcal{I}}_{p'})) \to \exists x((x \in {}^*\mathbb{N}') \wedge ({}^*\mathbf{q}(x,p') = y)).$$

The previous Theorems 2.1, 3.1, 4.1, 5.1 are all relative to a specific instruction paradigm and each holds for a collection of these instruction paradigms. Thus, the notion can be added and the additional statement that the results hold for each $n \in {}^*\mathbb{N}'$ and each $p \in [1,4]$. The special processes noted in the scheme in section 6 are applied to each set of instruction paradigms.

Each member of In Herrmann (2002), hyperfast propertons are mentioned as mediators for the automatic selection of a particlar member of ${}^*\mathcal{I}_p$. Certain not realized members of ${}^*\{\mathcal{I}\}_p$ are termed as covirtual. Notice that from the transformed formal statements above, there exist a member of ${}^*\underline{\mathcal{I}}_{p'}$ for each $\gamma \in {}^*\mathbb{N}' - \mathbb{N}'$. These can be used for various interpretations using either a q, q$'$, q type of sequence. Further, GGU-model predicted processes and entities can aid in comprehending the notion of the non-temporal and its relation to the temporal.

An important refinement to this participator model can be found in Herrmann (2014).

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