The Strengths of GD-model Attributes are not Uniquely Measurable Via Superstructure Nonstandard Analysis.

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Abstract: Since 1969, the superstructure has been the usual approach to nonstandard analysis. The method used to characterize a measure for the strength of a GD-model attribute that is comparable to a human attribute is, at least, partially expressible in terms of cardinality. In this paper, it is shown that for the non-atomic approach to superstructure construction such a cardinality does not have a set-theologic upper bound.

1. The GD-model.

This article is a theological application of the Grundlegend-Deductive Model (GD-model). This mathematical model predicts that Divine attributes, which are comparable to human attributes, are infinitely stronger than the corresponding human attributes. This article also contains some rather complex mathematics. This section is concerned with the basic superstructure approach to nonstandard analysis [2, 3, 6, 7].

The language L is represented by the set W$'$ or $\underline{\mathcal{W}}'$ in a standard superstructure $S(\mathcal{Q})$ based on the rational numbers $\mathcal{Q}$ as the set of atoms[2, 3]. (NOTE: The ground set can also be the natural numbers, integers, real numbers or other sets, as the case may be.) By a special coding [6, p. 57-58], any infinite set X is representable by a set Y of the same cardinality and which preserves the necessary atomic properties needed for the construction of a superstructure. Since Y is infinite, it contains a denumerable subset Q_Y. Hence, there is a bijection from $\mathcal{Q}$ onto Q_Y and this bijection is used to induce the structure of $\mathcal{Q}$ onto Q_Y. This induced structure implies that $\mathcal{Q}$ and Q_Y are isomorphic and have the same properties. Hence, Q_Y can be used as the set of rational numbers that also behave, with respect the superstructure, like atoms. There is a subset of $N_Y \subset Q_Y$ that behaves like the natural numbers $\mathbb{N}$.

In the previous article on measuring intelligence, the notion $|\cdots|$ is employed. This symbol is used to indicate the "power of a set" compared to another set, where $|A| < |B|$, means there is a injection A into B, but no injection on A onto B. And, $|A| = |B|$ means there is an injection on $|A|$ onto $|B|$ (i.e. a bijection). The order $\leq$ for the $|\cdots|$ has the same basic order properties as the ordering for the cardinalities of the sets [1, pp. 364-365]. In this paper, the order $\leq$ is denote by $\preceq$ [1] and the cardinality of a set is denoted by $\|\cdots\|$.

As first proposed, an alphabet is a finite set of symbols, images, and, by a coding, all human sensory information. The rational numbers can be considered as objects that are only members of a formal language. On the other hand, they can also be considered as members of an informal alphabet using notions such as Kleene tick notation. If they are part of our informal alphabet, it is trivial to consider the informal rational numbers as part of the informal language. This gives a denumerable alphabet. Each word is representable by a finite equivalence class $[\mathrm{f}] \in \underline{\mathcal{W}}'$of partial sequences and a unique $\mathrm{f}_n \in [\mathrm{f}]$, where each $\mathrm{f}_n(i)$ is an alphabet symbol and the finite $n > 0$ represents the length of a word as intuitively writing via the join operator. The length includes symbol recitations. With or without intuitive rational number symbols, the set W' or employed $\underline{\mathcal{W}}'$ is denumerable.

2. A Superstructure Approach.

In what follows, an altered superstructure approach is used [3]. In general, the superstructure ground set $\mathrm{X}_0 = \mathrm{W}' \cup \mathbb{R}$ (or $\mathbb{R}$ is replaced with $\mathbb{N}$ or the set of rational numbers $\mathbf{Q}$). As we are reminded by Robinson [6b, 90] and Mendelson [6b, p. 28], in accordance with the principles of mathematical modeling, abstract mathematial objects represent entities identified within other specific disciplines. The properties associated with various mathematical relations between the representative entities represent the behavior of the entities being so identified. The set W' represents the extended language concept as used in the intuitive model. Thus the language can contain symbols for each real number, an ordinary alphabet, and, additionally, images, diagrams and digital represented for virtual reality human sensory information. Which additional features one employed depends upon the application. The intuitive monoid behavior displayed by the basic justapostioning operator is reflected a monoid relation defined on W'.

It is shown in [6, p. 58-59], that atoms are not necessary for proper superstructure application to nonstandard analysis. Hence, this approach can be used for an infinite set X of the appropriate cardinality and for a superstructure, where the "ground set" Y of the same cardinality as X is obtained by a special coding of the members of X. The injection used for this coding is also used to impress upon Y any necessary structure defined for X or on subsets of X. For example, let $X = \mathrm{W}' \cup \mathbb{R}$. Then the special coding bijection $k\colon X \to Y$, yields disjoint $k[\mathrm{W}']$ and $k[\mathbb{R}]$. Further, all of the necessary relations and properties defined for W' and $\mathbb{R}$ are passed to Y via the bijection k. This process holds for the "language" portion of the of the foundational set X composed of the natural number, rational number or real number symbols as the case may be.

The set of equivalence classes as intuitively defined in [2] are represented here by the previous generic symbol $\underline{\mathcal{W}}'$. Thus, when one states that an image of a member of one of the equivalence classes in $\mathcal{W}'$ is an alphabet symbol "m," ones means that the image "represents" this symbol under the codings being employed. As with W', the intuitive monoid behavior displayed by the basic juxtaposition operator is reflected by a monoid relation defined on coded W' as well as on coded $\underline{\mathcal{W}}'$ [2b]. This coding statement will be understood. Under the rules for modeling, the intuitive language concepts can be directly associated with the coded entities.

Since there can be different X that yield the different Y, then rather then use the customary symbolism $^*Z$ to denote the monomorphism mapping applied to Z, $^{*_Y}Z$ is employed. This alteration in symbolism is used for $^\sigma$ as well. As usual, for the infinite set Y, the basic superstructures employed are $S(Y) = \bigcup\{X_i \mid i \in \mathbb{N}\}$, $X_0 = Y$, and $S(^{*_Y}Y)$, $X_0 = {}^{*_Y}Y$. Recall, that for any $X \in S(Y)$, $^{\sigma_Y}X = \{(^{*_Y}x \mid (x \in X) \wedge (x \in S(Y))\}$ and $^{\sigma_Y}$ embeds $S(Y)$ into $S(^{*_Y}Y)$ and the $^{\sigma_Y}$ objects model the standard superstructure $S(Y)$. Since this is an altered approach to superstructure construction, the $x \in S(Y)$ in the definition of $^{\sigma_Y}$ appears necessary. This allows the same identification as used in [2] to be applied to these superstructures. This yields that for $x \in X_0$, $^{\sigma_Y}x = \emptyset$.

As for this identification, since for ground sets Y and $^{*_Y}Y$, $a \in Y$ yields that $^*a \in {}^{*_Y}Y$, then it is customary to write $^*a = a$. Since in each case, a member of $\underline{\mathcal{W}}'_Y$ is a finite set of finitely many entities, and each of these entities is itself a finite set that reduces to finitely many members of the ground set, then this identification procedure also holds for $[f] \in \underline{\mathcal{W}}'_Y$. That is, for $^*[f] \in {}^{\sigma_Y}\underline{\mathcal{W}}'_Y$, $^*[f] = [f]$. Further, the relation $^{*_Y}{<_Y}$ and operation $^{*_Y}{+_Y}$ relative to $^{*_Y}N_Y$ are denoted as $<_Y$ and $+_Y$ since they can be considered as but extensions of those defined on N_Y. These identifications are used throughout the appropriate portions of what follows.

Notice that the definition of an hyperfinite interval such as $^{*_Y}[\mu,\nu]$, $\nu,\mu \in {}^{*_Y}N_Y$ under the identification is the same as $[\mu,\nu]$. Further, it is well know that, in general, for any $\nu \in {}^{*_Y}N_Y - N_Y$, and any $m \in N_Y$, $\{x \mid (m \leq_Y x) \wedge (x \in N_Y)\} \subset [m,\nu]$. This follows since for arbitrary $n \geq_Y m$, $n \in N_Y$, $[m,n] \cap (^{*_Y}N_Y - N_Y) = \emptyset$. Recall that the set $^{*_Y}N_Y - N_Y$ is termed as a set of "infinite" numbers.

Theorem 1. *For any infinite set X_0, the corresponding infinite sets Y and N_Y and a $\|S(Y)\|^+$-saturated model $^*\mathcal{M}$ contained in $S(^{*_Y}Y)$ imply that $\|S(Y)\| \leq \|^{*_Y}N_Y\| = \|A\|$, for any infinite internal set $A \in S(^{*_Y}Y)$.*

Proof. Consider infinite $Y = X_0 \in S(Y)$. From saturation, there exists a hyperfinite F_0 such that $^{\sigma_Y}X_0 \subset F_0 \subset {}^{*_Y}X_0$. (Note: Each hyperfinite sets is an internal set.) From [1], $X_0 \preceq {}^{\sigma_Y}X_0$. Since F_0 is hyperfinite, then there exists some $\nu_0 \in {}^{*_Y}N_Y - N_Y$ such that for the segment $[0,\nu_0]$, $[0,\nu_0] \simeq F_0$. Thus, $X_0 \preceq [0,\nu_0]$.

Now consider X_1. Then, in like manner, there exists some $\nu_1 \in {}^{*_Y}N_Y - N_Y$ such that $X_1 \preceq [0,\nu_1]$. Consider $[\nu_0 +_Y 1, \nu_0 +_Y \ \nu_1 +_Y 1]$. Then using $f(x) = x +_Y \nu_0 +_Y 1$, $0 \leq x \leq \nu_1$, $[0,\nu_1] \simeq [\nu_0 +_Y 1, \nu_0 +_Y \nu_1 +_Y 1] \succeq X_1$ and $[0,\nu_0] \cap [\nu_0 +_Y 1, \nu_0 + \nu_1 +_Y 1] = \emptyset$. Let $n_0 = 0$, $\nu_0 = m_0, n_1 = m_0 +_Y \ 1$, $m_1 = m_0 +_Y \ \nu_1 +_Y \ 1$. Then, since $m_0 <_Y n_1$, $[n_0,m_0] \cap [n_1,m_1] = \emptyset$. Suppose that for $k > 0$, $k \in \mathbb{N}$ there is a nonempty finite set of intervals $\{[n_i,m_i], \mid (0 \leq i \leq k) \wedge (i \in \mathbb{N}) \wedge (n_i \in {}^{*Y}N_Y - N_Y) \wedge (m_i \in {}^{*Y}N_Y - N_Y) \wedge (\forall j((j \in \mathbb{N}) \wedge (0 \leq j < i \leq k) \rightarrow (m_j <_Y n_i))\}$ and $[n_i,m_i] \succeq X_i,\ 0 \leq i \leq k$.

Consider X_{k+1}. Then there is a $\nu_{k+1} \in {}^{*Y}N_Y - N_Y$ such that $[0,\nu_{k+1}] \succeq X_{k+1}$. Let $n_{k+1} = m_k +_Y 1,\ m_{k+1} = m_k +_Y\ \nu_{k+1} +_Y 1$. Then $[n_{k+1},m_{k+1}] \succeq X_{k+1},\ m_j <_Y n_i,\ 0 \leq j < i \leq k+1,\ j,i \in \mathbb{N}$. Thus, $[n_p,m_p] \succeq X_p,\ 0 \leq p \leq k+1$ and $\{[n_p,m_p] \mid (0 \leq p \leq k+1) \wedge (p \in \mathbb{N})\}$ is a set of disjoint intervals. Hence, by induction, there is a set of disjoint intervals $\{[n_k,m_k] \mid k \in \mathbb{N}\}$ such that $[n_k,m_k] \succeq X_k,\ k \in \mathbb{N}$. Consequently, $\bigcup\{X_k \mid k \in \mathbb{N}\} = S(Y) \preceq \bigcup\{[n_k,m_k] \mid k \in \mathbb{N}\}$. (This comes from the fact that a surjection $f\colon A \rightarrow B$ reduces to an injection $g\colon A \rightarrow B$ and that the $[n_k,m_k]$ are disjoint.) But, $\bigcup\{[n_k,m_k] \mid k \in \mathbb{N}\} \subset {}^{*Y}N_Y$. Therefore, $S(Y) \preceq {}^{*Y}N_Y$. Hence from [1, Corollary 10, p. 365], $\|S(Y)\| \leq \|{}^{*Y}N_Y\|$. For any infinite internal set $A \in S({}^{*Y}Y)$, from [7, p. 38, 0.4.4], it follows that $\|S(Y)\| \leq \|{}^{*Y}N_Y\| = \|A\|$. (Considering [1], I can find no reason why this statement is not valid for a superstructure constructed using the coded method presented in [6].) This completes the proof.

From Theorem 1, $\|S(Y)\| \leq \|{}^{*Y}N_Y\| = \|{}^{*Y}Q_Y\| = \|{}^{*Y}Y\| = \|{}^{*Y}\mathbf{L}_Y\|$. Thus, the cardinality of each ${}^{*Y}\mathbf{L}_Y$ is rather "large" compared to $\|\mathbf{L}_Y\|$. Further, $\|N_Y\| \leq \|Y\| < \|S(Y)\| \leq \|{}^{*Y}N_Y\|$.

The language L, the rule of inference PR and all other standard entities are modeled in $S(Y)$ by $\mathbf{L}_Y$ and $\mathbf{PR_Y}$ **etc.** Further, in general, the attributes and the "greater than" order are also modeled by members of $S(Y)$ [2, Section 4.4]. For $\mathbf{L}_Y,\ \|\mathbf{L}_Y\| = \|N_Y\| = \|Q_Y\| = \|\mathcal{E}_Y\| = \|\mathbf{PR_Y}\|$, etc.

Let $N_Y \subset Y$ and $N_{Y_1} \subset S({}^{*Y}Y) = Y_1$. Then $\|S(Y)\| \leq \|{}^{*Y}N_Y\| < \|S(Y_1)\| \leq \|{}^{*Y_1}N_{Y_1}\|$. If one considers cardinalities as a measure of a type of "size," then nonstandard ${}^{*Y_1}N_{Y_1}$ is considerably "greater in size" than ${}^{*Y}Y$.

For a given infinite Y, the higher-language is ${}^{*Y}\mathbf{L}_Y$ and $\|{}^{*Y}\mathbf{L}_Y\| = \|{}^{*Y}N_L\|$. Thus, given any such higher-language ${}^{*Y}\mathbf{L}_Y$, there exists a higher-language ${}^{*Y_1}\mathbf{L}_{Y_1}$ such that $\|{}^{*Y}\mathbf{L}_Y\| < \|{}^{*Y_1}\mathbf{L}_{Y_1}\|$. Consequently, for superstructure obtained higher-languages, there is no upper bound in the terms of cardinality.

In the proof of Theorem 4.4.1 [2], b is a basic attribute such as "intelligent." A member of C_b is the coded form of "very, very, . . . , b." It is shown in the proof that for any superstructure $S(Y)$ and a corresponding attribute **b** and any $\nu \in {}^{*Y}N_Y - N_Y$ there is an ultraword c, a **higher-attribute**, such that c is greater than or better than (i.e. ${}^{*Y}<_B$) any ${}^{*Y}w \in {}^{\sigma Y}C_b$. Under the identification $w \in C_b$.

Due to the construction of $\underline{\mathcal{W}}'_Y$, the form of this $c \in {}^{*Y}\underline{\mathcal{W}}'{}_Y - \underline{\mathcal{W}}'_Y$ can be determined. (Note: When the notation $\underline{\mathcal{W}}'_Y$ and elements are considered, the original intuitive injection i is supressed and understood relative to the set of equivalence classes employed.) In particular, $c = [g]$ and each member of $[g]$ is an internal function. There is a unique function $f \in [g]$ and $\nu \in {}^{*Y}N_Y - N_Y$ such that $f\colon [0,\nu] \to {}^*T$, ${}^*T = {}^{*Y}N_Y$ and $f(0) = \mathrm{b} \in \underline{\mathcal{W}}'_Y$, $f(j) = \mathrm{very},|||$, where $1 \leq j \leq \nu$. This follows since, for each $0 \leq n \in N_Y$, there is a $w = [h] \in C_b$, and a unique $k \in [h]$ such that $k\colon [0,n] \to T$, $k(0) = \mathrm{b}$ and $k(j) = \mathrm{very},|||)$, where $1 \leq j \leq n$. These unique functions, when restricted to $[1,\nu]$ and $[1,n]$, respectively, are the "counting" functions, where the number of embedded "very," strings is directly related to the "size" of the intervals $[1,n]$ and $[1,\nu]$. The cardinality of $[1,n]$ can be symbolized as "n." And, if $n = 0$, then $[1,n] = \emptyset$ and $\|[1,n]\| = 0$.

More directly, there is no bijection Θ from any set of the form $[1,\nu]$ onto $[1,n]$. For if there is, then since internal $[1,n] \subset [1,\nu] \neq [1,n]$ the restriction $\Theta|[1,n]$ is an injection on $[1,n]$ onto a proper subset of $[1,n]$. However, no such mapping exists [8]. Consequently, $[1,n] \prec [1,\nu]$ and $\|[1,n]\| < \|[1,\nu]\| = \|{}^{*Y}N_Y\| = \|{}^{*Y}L_Y\| > \|S(Y)\|$ since $[1,\nu]$ is infinite. This gives additional guidance since ${}^*{<_B}$, the greater than ordering, only states that $w\,{}^*{<_B}\,c$ since $n < \nu$. This applies to an attribute b associated with any biological entity in any universe (even a countably infinite collection of universes with the attribute as a combined attribute) that can be qualified by the "very," strengthening. There is, however, a problem with using the cardinality within the same structure as a measure for the strength of an attribute.

Within $S({}^{*Y}Y)$, there are two (and many more) higher-attributes c_1 and c_2 in the language ${}^{*Y}\mathbf{L}_Y$ such that for each $w \in C_b$, $w\ {}^{*Y}{<_B}\ c_1{}^{*Y}{<_B}\ c_2$. For the ordering ${}^{*Y}{<_B}$, the c_1 attribute has the measure ν for the segment $[1,\nu]$ and c_2 has the measure $\mu > \nu$ and μ corresponds to $[1,\mu]$. However, $\|[1,\nu]\| = \|[1,\mu]\| = \|{}^{*Y}N_Y\|$ since $[1,\nu]$ and $[1,\mu]$ are infinite and internal hyperfinite sets. Hence, within the $S({}^{*Y}Y)$, it is only necessary to consider one of these infinite measures for the results stated in [2].

External cardinalities, within a specific structure $S({}^{*Y}Y)$, do not have the same properties as the greater than ordering, ${}^{*Y}{<_Y}$. For any structure $S({}^{*Y}Y)$, there is no $\nu \in {}^{*Y}N_Y$ that is a ${}^{*Y}{<_Y}$ upper bound. But, such subtle words as c_ν, c_μ exist for each $\nu, \mu \in {}^{*Y}N_Y - N_Y$. So, in this case, using **any** $\nu \in {}^{*Y}N_Y - N_Y$, the ${}^*{<_\mathrm{B}}$ has no upper bound within $S({}^{*Y}Y)$. But, when ν is replaced by $[1,\nu]$, then, viewed externally, $\|[1,\nu]\|$ is an upper bound relative to the structure $S({}^{*Y}Y)$. From this, one might concluded, that there is an external upper bound.

For arbitrary superstructure $S(Y)$, applying the above procedure to the superstructure $S(Y_1)$, it follows that for $\nu \in {}^{*Y}N_Y - N_Y$ and $\mu \in {}^{*Y_1}N_{Y_1} - N_{Y_1}$ that $\|[1,\nu]\| < \|[1,\mu]\|$. Unless the order ${}^{*Y}{<_Y}$ is extended in some way, external cardinalities appear to be an appropriate way to compare such ν taken from two such distinct structures. Since $S(Y)$ is arbitrary, then no matter how a superstructure's ground set Y is obtained there is another superstructure that verifies that there is no upper bound for $\|[1,\nu]\|$, where ν is an infinite number. Thus the "greater than" ("better than") ordering ${}^*\!<_{\rm B}$, as it relates to the external cardinality, also has no upper bound that can represent the strength of a higher-Divine-attribute for the class of all superstructures thus far being considered. This follows since for every infinite number ν in any superstructure, ${}^*C_b$ contains an ultraword c that contains $\|[1,\nu]\|$ "very, $|||$" symbol-strings.

Thus, for various none-abstract models, using the statement that a higher-attribute is "infinitely greater than" the same attribute displayed by an entity in the sense of the greater than ordering is but a "partial" set-theoretic measure for the strength of the higher-attribute. Stating that a specific higher-attribute is "infinitely greater than" a similar attribute should also be considered as generic in character. The class of all such characterizing cardinalities can be considered as a type of generic "ultimate infinite" concept. To maintain consistency, simply consider this generic "ultimate infinite" to be "greater than" all of the partial measures used. This "greater than" notion is external to the set-theory being applied. It is obtained through a single application of scientific induction.

Also it is demonstrated, for the methods I use to obtain an higher-language, that the cardinality measure gives but a partial measure as to the "size" of an higher-language. There is no set-theoretic upper bound for this size notion when additonal superstructures are considered. Thus, stating that an ultimate higher-language is "infinitely greater than" any natural language should be considered as a generic statement.

The above discussion can be applied to the rules for logical deduction. Notice that a formal proof can be represented in $\rm W'$ by a single finitely long word $\rm w \in W'$. The number "n" of steps in a formal proof, or more generally as produced by the deduction algorithm, is a measure for the number of deductions. This number is determined by a specific member of the equivalence class of partial sequences associated with $\rm w \in W'$. It is known that, at the least for a formal predicate language, there exists formal theorems that for any fixed $n \in \mathbb{N}$ require, at least, n steps. Considering propositional deduction relative to $\rm W'$ and properly characterizing this fact leads to the prediction that, for any $\nu, \rho \in {}^{*Y}\!N_Y - N_Y$, there is higher-form of deduction characterized by the "step number" ν.

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