What do the Expressions - Space, Space-Time, Time-Dilation and an Expanding Universe - Actually Signify?

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*. . . we are the victims of picture-thinking**

A goal of this article is to show explicitly that the "geometric" language used in the classical General Theory of Relativity (GR) does not usually correspond to configurations as geometrically described by Riemannian Geometry (RG). The geometric language used in RG often corresponds to but an analogue model for behavior and mostly does not, in its actual form, directly correspond to reality. In GR, what such terms as space, space-time (spacetime), time-dilation and the expansion of space actually signify is discussed. It is shown how these terms relate to physical entities. Some mathematical expressions are used. However, any deep comprehension is not necessary. For the more complex expressions, simply ignore their appearance since their basic significance is intuitively discussed.

Physical Space

For this theory, the actual notion of "physical space" is not as it is often pictured in the popular literature.

> The further implication is, therefore, that physical space is not simply a mathematical abstraction which it is convenient to employ when considering distance relations between bodies, but exists in its own right as a separate entity with sufficient internal structure to permit the definition of inertial frames. However, all available evidence suggests that space cannot be defined except in terms of distance measures between physical bodies. . . . Physical space is then, nothing more than the aggregate of all possible coordinate frames [7, p. 127]

The notion of coordinate frames (i.e. systems) is a pure imaginary concept. They do not exist as nature-systems. Hence, "space," in this definition, is not composed of real physical entities. Lawden's definition of space is adequate for many applications but needs revision when the notion of an "expanding space" is considered. This revision does not alter the basic model-theoretic facts discussed. Indeed, as quoted below, world authorities in GR agree that various notions used within GR only yield an analogue model for behavior. (I have underlined and marked with numbers, which are not in the original, a few portions for further discussion.)

If "physical space" is not composed of some sort of known physical entities under the Lawden's definition, then how does "geometry" enter into this subject? One way of explaining this is also given by Lawden when he "defines" a "very general type of frame."

* C. S. Lewis, Miracles, Macmillan Publishing, N. Y. (1978), p. 31.

(1) Imagine that the whole of the cosmos is filled by a fluid whose motion is arbitrary but non-turbulent (i.e. particles of the fluid which are initially close together, remain in proximity to one another). Let (2) each molecule of fluid be a clock which runs smoothly, but not necessarily at a constant rate as judged by a standard atomic clock. . . . Each clock will be allocated three spatial coordinates ξ^1, ξ^2, ξ^3 according to any scheme which ensures that the coordinates of adjacent clocks only (3) differ infinitesimally. These coordinates ξ^α of a clock will be supposed never to change. Any event taking place anywhere in the cosmos can now be allocated unique space-time coordinate ξ^i $(i = 1, 2, 3, 4)$ as follows: (ξ^1, ξ^2, ξ^3) are spatial coordinates belonging to the clock which happens to be adjacent to the event when it occurs, and ξ^4 is the time shown on this clock at this instant [7, pp. 131-132].

Thus far, statement (1) should immediately indicate that this is not a true concrete physical model. Phrase (2) has little physical meaning except for possible alterations in physical behavior. It is used here to correspond to the notion of a "clock" located at every "position" within our universe. As will be seen (statements (6) and (8) below), in (3) Lawden means the actual infinitesimal notion. This is related to the long-standing problem (solved in more than one way [6]) that attempts to correspond all aspects of the infinitesimal calculus to actual physical-like objects. However, I point out that this can be the case for subparticles [3,4,5] as used in the General Grand Unification model. Please note that the imagined particles of fluid are probably physically contained within "something" but are only associated with the imagined space-time as defined by Lawden. I note that there are more abstract and "modern" definitions for space-time. But, they tend to confuse the issue via abstractions. For example, to some, space-time is a set of all events. This definition is then further refined so as to relate these events to objects within a differential manifold. I need not continue this aspect since such abstractions usually come from combinations of basic and simple notions. Eventually, all such abstractions relative to the physical behavior are reduced to the local Euclidean world in which we dwell. All that is needed for my demonstrations is classical RG and GR.

We shall further generalize the coordinates allocated to an event. Let x^i $(i = 1, 2, 3, 4)$ be (4) functions of the ξ^i such that, to each set of values of the ξ^i there corresponds one set of values x^i, and conversely. We shall write

$$x^i = x^i(\xi^1, \xi^2, \xi^3, \xi^4) \qquad (45.3)(1)$$

Then the x^i, also, will be accepted as coordinates, with respect to a new frame of reference, of the events whose coordinates where previously taken to be the ξ^i. . . . All possible events will now be mapped upon (5) a space

$\mathcal{S}_4$, so that each event is (6) represented by a point of the space and the x^i will be the coordinates of this point with respect to a coordinate frame. $\mathcal{S}_4$ will be referred to as the *space-time* *(6) continuum* [7, p. 132]. (The term "mapped," in this case, can be considered as a mental correspondence between the events and the "coordinate space" $\mathcal{S}_4$. The term can be more formally defined.)

This last description shows that events that occur in some sort of intuitive but undefined physical space are replaced by a mathematical representation that exists only in an imagined mathematical "space" of coordinates. Of great significance is the fact that the model only has the ability to describe the behavior of events as this behavior is measured by a specific set of coordinates $\mathcal{S}_4$. The pure mathematical objects in (4) are also called one-to-one functions and they generate infinitely many such $\mathcal{S}_4$. One of these would correspond to a rectangular coordinate system and, under this definition, each pair of $\mathcal{S}_4$ are related, at least, by a one-to-one correspondence. The word in (5) refers to the pure mathematical sets $\mathcal{S}_4$ characterized by (6). The term "continuum" used in (6) is very important for field theories. It refers to the values of the coordinates and this property is a classical property that is needed in RG modeling where the language of infinitesimals is used. In order to use the classical Calculus and Absolute Calculus (Tensors) as analytical tools, this is required. However, to be absolutely correct the set $\mathcal{S}_4$ should be extended to the infinitesimal world as the next quotation shows.

If, for one such observer, the events at the points having rectangular Cartesian coordinate (7) $(x, y, z)(x + dx, y + dy, z + dz)$ occur at the time (8) $t, t + dt$ respectively, then. . . . The *interval* between events ds will be defined by

$$ds^2 = -c^2 d\tau^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \qquad (45.5)(2)$$

The coordinate (x, y, z, t). . . will be related to the coordinates x^i defined earlier, by equations

$$x = x(x^1, x^2, x^3, x^4), \text{ etc.} \qquad (45.6)(3)$$

and hence

$$dx = \frac{\partial x}{\partial x^i} dx^i, \text{ etc.} \qquad (45.7)(4)$$

Substituting for dx, dy, dz, dt in equation (2), we obtain the result

$$ds^2 = g_{ij} dx^i dx^j \qquad (45.8)(5)$$

. . . . The space-time continuum can accordingly be treated as a Riemannian space with the metric given by equation (45.8) [7, p. 132-133].

(Note: Whenever a term in an equation contains super or subscripts (indexes) that are repeated, then this is taken to mean that you sum these terms over the entire collection of natural number over which the indexes independently vary. In this case, i, j vary independently over the numbers 1,2,3,4).

From a simple viewpoint, GR is an interpretation of an RG restricted investigation of coordinate (or component) space, where classically and intuitively such a term as "n-dimensions" (n-D) indicates the number of independent slots or positions in the expression $(\cdot, \cdot, \ldots, \cdot)$. The equations (3) are coordinate transformations that, under the necessary constraints, yield another coordinate space and these two coordinate spaces may be compared by viewing them from a standard coordinate system. Moreover, throughout this article, I discuss significant aspects of GR that can be obtained without RG [4]. (Due to the method used to derive the line-element, the dS^2 in [4] is the $-ds^2$ of (2).) Indeed, the basic entities need not refer to any geometric language whatsoever. This gives additional evidence that the model is but "analogue in character." I also point out that the terms line-element and element of length are used for expressions such as (5). The physics community calls it a "metric." *The metric* (2) *is a physical statement that reveals the required electromagnetic propagation properties as measured by an infinitesimal light-clock not affected by a potential velocity. The four dimensions are needed due to how velocity is defined.* [4, 56-57]. This interval is called the infinitesimal *Minkowski* or *chronotopic* interval. The second coordinate expressions in (7) and (8) show that the model does not correspond to physical entities within our universe. (Below, I'll discuss some aspects of the theory of subparticles [3,4,5], where subparticles should never be viewed in any manner as particles or seriously imaged in any mental geometric form. Combinations of subparticles within a substratum world can be associated with infinitesimal coordinate measures.)

Notice that equations (4) state that the functions x are not "any" selected functions but they must have special mathematical properties for ds^2 to exist as a mathematical object. For GR gravitational field theory and our universe, the g_{ij} are not, in general, the values as stated in (4), but rather they must satisfy in one way or another the Hilbert-Einstein gravitational field equations relative to a specific coordinate system. As mentioned, the symbols in (5) are an abbreviation for a finite sum of the terms $g_{ij}dx^i dx^j$ as the i, j vary independently over the numbers 1,2,3,4. The language of RG does not, as yet, refer to real physical stuff. Since space is not defined in terms of any physical material, such expressions as "the curvature of space-time" can only have meaning for physical behavior - behavior that is essentially controlled by equations such as (5). *For GR, what is often called space-time is coordinate space* $\mathcal{S}_4$, *where the interpreted physical behavior is controlled by the* g_{ij} *that appear in a specify metric. Indeed, even expressions like* (5) *are termed as space-time.*

Riemannian Geometry and Gravitational Fields

I will not discuss what some very imaginative particles physicist consider as the actually entities that comprise a gravitational field. Since the objects investigated are not predicted to exist and, apparently, cannot even be indirectly detected, I consider such a discussion as the rankest form of speculation that displays nothing more than the prideful nature of humankind. It will be shown for gravitational fields that, in many cases, the geometric notions are substitutions for the still mysterious notion of gravitational "forces."

Are there individuals who have specialized in GR and who also more truthfully state the RG is but an analogue model for behavior?

> What is the substance out of which the universe is made? . . . . Riemannian geometry likewise provides a beautiful vision of reality . . . to see in what ways geometry is inadequate to serve as primordial building material . . . (1) geometry is as far from giving an understanding of space as "elasticity" is from giving understanding of a solid [10].

Elasticity is a property of a solid relative to its behavior under certain circumstances. Thus, Patton and Wheeler admit publicly that RG is but an analogue model for behavior within a gravitational field. Thus far, the actual GR theory relates abstract mathematical entities that require interpretation. I firmly believe that great confusion and, indeed, error occurs when one attempts to picture certain notions such as curvature for a 4-D space-time via some form of physical diagram. The confusion results from the fact that it does have intuitive geometric meaning in a 3-D world.

Many of our human experiences are relative to observations via rectangular and equal interval systems, at least, locally. Intuitively, geometry is defined as the science that studies the shape and size of things. In technical areas, it is often the study of the invariant properties of the given elements under specific groups of transformations. The term "curvature" comes from notions in classical RG about 3-D spatial properties of a curve or surface and such properties do correspond directly to our perceived ideas of shape, size and measurement. RG is also known as differential geometry where the differential calculus or even the tensor calculus is used to study general properties of curves and surfaces. I have published in nonstandard differential geometry and the Special and General theories of relativity, but the results of these researches are beyond the scope of this article.

Within RG, a *surface* is defined by three equations $x^i = f^i(u^1, u^2),\ i = 1, 2, 3$, with certain additional properties. The coordinates (x^1, x^2, x^3) are considered as representing points in a 3-D rectangular (Cartesian) coordinate system. (Such a purely 3-D view, usually, should not be used when RG is applied to GR.) The (u^1, u^2) are usually restricted to a region R that is viewed in an u^1u^2-rectangular coordinate system for comparison purposes. Intuitively, the "mapping" (i.e. the three equations) takes the region R and maps it into the 3-D space (i.e. corresponds each point in R with a point in 3-D space (x^1, x^2, x^3)) so that it now takes the form of a surface. A curve on the surface is defined by two equations $u^1 = g^1(w),\ u^2 = g^2(w)$, where w might vary over an interval I of real numbers [1, p. 123]. Notice that in the 2-D region R, these two equations can be graphed as a curve in the region (i.e. the interval I has been mapped or changed into a curve in R). By substitution, the curve can be viewed from 3-D space as a curve, restricted to a surface. Indeed, the curve is but the result of two mappings that start with I and yield a restricted 3-D space curve. The maps simply take the interval and distort it into a 3-D curve.

But, how does this relate to the gravitational field equations? Although known by very few, these equations "need to be viewed" from a 5-D coordinate system and, usually, this fact is never mentioned. Below I discuss why, basically, this extra dimension is needed. When classically discussing the idea of an expanding universe an additional "surface" coordinate is used. To view this space, one would need a 6-D space. *One reason for not mentioning this is that, when dealing with a 2-D surface, certain defined geometric properties that characterize a specific surface are first viewed from a 3-D space and then expressed in terms of the* u^1, u^2*-coordinate values.* However, for geometry, the f^i are needed in order to obtain the actual g_{ij} which will be expressed in terms of the u^1 and u^2 values. Only in the case that these *metrical coefficients* g_{ij} can be found by other means will the original equations f^i be unnecessary for their determination. *However, this does not alter the original and required geometric definitions.* It is becoming clear that using notions from RG to discuss behavior within a gravitational field may be but an re-interpretation for the original geometric notions and appears to yield a mere analogue model for behavior. Although geometric language may be maintained, usually with additional prefixes such as "hyper," any physical meanings given to most of the geometric terms do not refer to the originally defined geometric notions. (Terms such as "volume," that use the Euclidian units, usually have their ordinary intuitive meanings.)

Recall, that, in what follows, one need not know what the symbol signifies mathematically. If the mathematical space and the functions used are of a specific type, you may have points on a 2-D surface, in 3-D space, connected by a curve called a geodesic. In RG, an expression that governs the behavior of this curve is that the curves intrinsic equations (i.e. in terms of "s") satisfy

$$\frac{d^2u^i}{ds^2} + \left\{ \begin{matrix} i \\ jk \end{matrix} \right\} \frac{du^j}{ds}\frac{du^k}{ds} = 0,\ i, j, k \in \{1, 2\} \tag{6}$$

where the Christoffel symbol of the second kind $\left\{ \begin{matrix} i \\ jk \end{matrix} \right\}$ is totally determined by the g_{ij} expression for the surface in which the curve is contained. The g_{ij} can be expressed as the coefficients of an expression that takes the same form as (5).

How is RG used to model behavior in a gravitational field? A *test particle* is supposed to be an actual physical object with an extremely small mass. This very small mass has an associated gravitational field. However, this field is assumed to be of such a miniscule strength that it will not measurably affect the gravitational field that generates the particle's motion. The particle's behavior is modeled by relations between coordinates (x^1, x^2, x^3, x^4). The following is an example of how the model is constructed for a gravitational field, where the field affects events. (Note: ds is the "interval" in what follows.)

> Let (x^1, x^2, x^3, x^4) be the coordinates of an event in this frame. The interval between to contiguous events will then be given by equation (45.8). If an observer using this frame releases a test particle and observes its motion relative to the frame, he will note it is not uniform or even rectangular and will be able to account for this fact by assuming the presence of a gravitational field. He will find that the particle's equations of motion are
>
> $$\frac{d^2x^i}{ds^2} + \left\{ \begin{matrix} i \\ jk \end{matrix} \right\} \frac{dx^j}{ds}\frac{dx^k}{ds} = 0 \qquad (46.5)(7)$$
>
> [7,p. 134].

Equations (7) are of the exact same form as (6) but $i, j, k \in \{1, 2, 3, 4\}$. Further, Lawden gives the usual argument relative to such metrics as (2) and neighborhoods of a point that the quantities g_{ij} can be taken as satisfying the gravitational field equations. As Lawden states, "This means that the g_{ij} determine, and are determined by, the gravitational field" [7, p. 134]. Solutions for (7) require the constant initial conditions $dx^i/ds,\ x^i(s),\ i = 1, \ldots, 4$. If you consider (6) where there is no gravitational field present, then (7) reduces to the constant linear-type motion characterized by $d^2x^i/ds^2 = 0,\ i = 1, \ldots, 4$. *Hence,* (7) *states how, under these constant initial conditions, a test particle's behavior is only altered from this linear-type behavior by a gravitational field. This is the meaning of the expression freely falling.* Technically, comprehension of paths of motion requires graphing the paths as viewed from a 5-D coordinate system, a rather difficult thing to do. On the other hand, holding one parameter as a constant, one might consider them as they would be graphed in a 4-D coordinate system, still a difficult thing to do, unless one holds other parameters constant. This is exactly what is done. One or more parameters are held constant or, at least, not considered as part of the coordinate system. When we graph a path of motion in 3-D space and it varies in time, then the mappings are used and the time "interval" is often considered as part of an auxiliary 1-D system.

In RG, when viewed from a 3-D space, the geodesics on a right cylinder are helices. But, geodesics on this "curved" surface have the same metric properties as lines in a plane.

One might say that when viewed from the 2-D surface, they appear to be lines. For higher dimensions, the same conclusions hold. Thus, one must be careful when describing such behavior. The fact that the language of "curvature" is used comes from the mathematically rigorous notion that a 4-D surface may have the same metrical properties that characterize curvature for a 2-D surface viewed from 3-D space. In particular, if one such numerically valued expression is identical to zero, then the surface has the metrical characteristics for a plane. Thus has developed the misguided terminology that gravitational fields and "curved space-time" depict exactly the same physical notions. But, at least for Lawden, space-time is not a physical entity. Indeed, even for subparticle theory space-time is not curved. For physical behavior, such notions should be expressed in physical terms. Can this be convincingly demonstrated?

What I briefly discuss was will known and accepted in the 1920s and 1930s. This was before society was inundated by the mental pollution fostered by modern science-fiction enterprises. The notion of the curvature of space-time produced by a gravitational field is a substitution for gravitational forces. Simply take expression (7) and multiply it by the actual small mass, m, of the test particle. One gets

$$m\frac{d^2x^i}{ds^2} + m\left\{ \begin{matrix} i \\ jk \end{matrix} \right\} \frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

or

$$m\frac{d^2x^i}{ds^2} = -m\left\{ \begin{matrix} i \\ jk \end{matrix} \right\} \frac{dx^j}{ds}\frac{dx^k}{ds} = F_i, \tag{8}$$

where it should be mentioned that one of the forces is related to x^4. However, if x^4 is interpreted as "time," then, as well be shown, this can be interpreted as but a "force" that alters devices that measure "time." Recall, that the actual reason that the measurements of both time and distance are required is due to the methods used to measure speed or velocity as it is affected by a gravitational field.

In 1915, David Hilbert presented his derived gravitational field equations - equations that include a type of unification for gravity and electromagnetism. This was followed five days later when Einstein presented essentially the same gravitational portion of these Hilbert equations without derivation but rather through the "guess" process [8, pp. 24-34]. At that time, geodesic equations were usually derived by a different method using the famous Euler equations in terms of a type of time parameter ξ. When such a derivation is completed [1, pp. 177-178, where $\xi = t$] a unique path for a freely following particle satisfies the equations

$$m\frac{d^2x^i}{d\xi^2} = F'_i. \tag{9}$$

The F'_i components are generated by a slightly different expression than the F_i. Obviously, (9) can be interpreted in terms of forces (i.e. $F = ma$). In the force interpretation, these particle paths are similar to the lines of force surrounding various magnetic sources. Indeed, when gravitational fields are illustrated for a material body, what is usually illustrated are specially selected geodesics. Hence, rather than stating that particles follow curved paths due to the curvature of space-time, when viewed from a local coordinate system, one can state that gravitational forces cause the particle's behavior. The same force notion is also used for other behavior stated in terms of curvature such as the Riemann-Christoffel "curvature" tensor. These results are extended to all physical entities including any massless ones, where they are analyzed using expressions different from (7). Although what actually produces such force effects may, at present, not be fully known, the difference between a space-time curvature statement and the force statement is that humankind does "feel" and have experiences with gravitational forces as well as many others. *Much physical behavior that is altered by gravitational fields can be correctly described in terms of the ordinary physical language of forces, stresses and the like.* It should be clear that such notions as the curvature of space-time have no physical meaning in terms of the geometric definition unless one physically interprets the notion in terms of how physical entities behave as compared to their behavior in a local space, say characterized by (2), a space without gravitational fields. Indeed, modifying (2) in order to obtain gravitational field properties, yields direct evidence that gravity and electromagnetic propagation properties are inseparably related.

RG and GR Interpretation Methods

Suppose that in 1600 AD human beings on planet Earth were not capable of mathematically modeling gravitational behavior but they did have a language that they used to describe physical behavior. A scientist lets a small object drop unimpeded from various windows in a tower so that the object lands in a pan of loose soil. He makes notes that the further the object falls the more dust is kicked-up when the object hits the soil. He knows from other experiments that he gets the same changes in the dust patterns by throwing the same object from a fixed position but "faster" and "faster." He knows that almost all individuals with whom he cares to communicate have a common understanding of terms such as "further," "faster," "towards," "falls," and the like. He describes the object's behavior in terms of his observations. He states, "When it is dropped, the object seems to fall faster and faster towards the pan of soil the further and further it is from the pan." He then allows 150 other individuals observe the same motion and the additionally needed experiment and 148 agree that his description is correct. He then generalizes and states that all individuals that have the same observational abilities as the 148 that agreed with him will observe this same behavior. Empirical physical science associates with such experiments a basic rule that, even in the absence of such observations, the object will behave in the same described manner. He, of course, does use intuitive modes of measurement to arrive at his description.

To be more "scientific," the scientist constructs a "clock" and uses a ruler to measure the distance the body falls via the window heights. He uses his clock to measure the time of fall. He spends twenty or more years trying to find a relation between the different window heights and his clock measurements. Finally, he changes the rules he is supposed to use and concludes that the distance the object falls approximately varies directly as the square of the time and interestingly he contradicts the basic physical science of his time for objects of different weights (masses) and shows that the weight of the object does not alter this relation significantly. (Of course, the constant of proportionality balances the units.) Using the Merton rules for speed determination, he shows that the square of the speed is approximately proportional to the distance the body falls. But, then a clever inventor builds another clock called a differential clock, a clock that although not appearing obvious is related to the distance as marked on the tower. The scientist then measures the time and now discovers that the distance the object falls approximately varies directly as the time and calculates that the speed is approximately constant. This second set of measurements contradicts the basic description for the object's behavior. The basic description was not dependent upon the so-called scientific measurements. Probably, his observation should not change simply because he has "changed his mind" by using the differential clock. Indeed, he might conclude that the differential clock is a "doctored" clock used to measure this local basic physical behavior. The scientist is further invigorated by his revolutionary discovery that time, in general, can be considered as an independent parameter since this is how he developed the mathematical model that predicts the observation. This embellished partially true story is significant for what follows.

It is instructive to investigate more deeply the relation between RG and its application to GR. The actual methods used within GR to investigate test particle behavior are not equivalent to those used within classical tensor analysis as applied to RG. Due to the restricted types of coordinate transformations required, one aspect of RG states that a tensor equation does not change its form when transformed by any member of this collection of coordinate transformations. All tensor equations can be rewritten as a tensor expression that equals a zero tensor. For a zero tensor, all of the tensor components must be zero. When using a proper transformation, the actual transformed components of any zero tensor satisfy theorem [18.1] in [1, p. 91]. This theorem states that if all of the components are zero for one coordinate space, then they will also be zero when calculated for any proper coordinate transformation. (The term "proper" refers to a specific technical requirement called "regularity.") This is a mathematical fact. From the 3-D geometry view, let $x^i = f^i(u^1, u^2),\ i = 1, 2, 3$ be the surface equations with their special partial derivative properties at each point (u^1, u^2) of a domain. To obtain a graph for the curve in 3-D space, we have $u^1 = g^1(w),\ u^2 = g^2(w)$. The Euclidean metric in 3-D space is $ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$. Then $dx^i = \frac{\partial x^i}{\partial u^1} du^1 + \frac{\partial x^i}{\partial u^2} du^2,\ i = 1, 2, 3$. (You don't need to remember how this expression is obtained or even what it means.) Now if you substitute this last expression into the ds^2 and us a little algebraic manipulation, you get the following expression that is exactly the same as (5)

$$ds^2 = g_{11}(du^1)^2 + g_{12}du^1du^2 + g_{21}du^2du^1 + g_{22}(du^2)^2. \tag{10}$$

In this case, the

$$g_{\alpha\beta} = \frac{\partial x^i}{du^\alpha}\frac{dx^i}{u^\beta}\left(= \sum_{i=1}^{3}\frac{\partial x^i}{du^\alpha}\frac{dx^i}{du^\beta}\right),\ \alpha, \beta \in \{1, 2\},\ g_{12} = g_{21}. \tag{11}$$

As introduced by Gauss, (10) is called the *first fundamental form*. The ds is used as a measure of distance on the surface. Distance, angles between surface curves viewed as angles between tangents, and elements of area can be expressed in terms of the $g_{\alpha\beta}$. (However, technically, distance, the angles and the like are as they would be measured in terms of 3-D space and Euclidean units.) Now if you make a proper surface coordinate transformation and transform the formula and the coordinate names used, then you get the same numerical values. From this point of view, basic geometric information that characterizes the surface remains fixed. Is it necessary to investigate these surface properties from the 3-D viewpoint?

"However, once the formulas for the measurements of length, angles, and area have been found in terms of the first fundamental form (equation (10)) of the surface, thereafter these metric formulas may be used without considering the surface as embedded in space" [1, p. 146]. Yes, you may do so. *But, this does not eliminate the original geometric definitions.* These properties are called *intrinsic* since they can be expressed in terms of the coefficients in (10). If for a specific coordinate system, the form (10) is identical for two surfaces (they are *isometric*), then as far as the measurements of these intrinsic properties are concerned there is no difference in these two surfaces from the differential geometry viewpoint although physically there may be other factors that make the surfaces very different. Indeed, "a cylinder and a cone are isometric to a plane" [1, p. 147]. One important result relative to this is that two surfaces are isometric to a plane if and only if the Riemann curvature tensor $R_{ijkl} = 0$, where 0 denotes a zero tensor. No proper coordinate transformation will alter the form of this equation.

But, distinct from the geometry, GR tensor equations, in a general manner, state relations between physical properties that, when interpreted, are not merely geometric. These properties are hidden within the tensor expressions and, based upon the methods used to interpret physical behavior, it appears that different behaviors are exhibited by different coordinate transformations. There is a reason for this, but unfortunately in applications to physical behavior it has been claimed that the invariance of these geometric measures under coordinate transformations must hold for all physical behavior as well. This claim is used to establish some significant results (i.e. Birkhoff's Theorem for example). A few examples that show that this claim is false, in general, is all that is needed.

Transform the spatial part of (2) using the standard spherical coordinate transformations and obtain the expression for ds^2 without the presence of a gravitational field. If a type of velocity (the potential velocity) v_p influences behavior in a direct and simple manner, then (2) reduces to

$$dS^2 = -ds^2 = c^2 d\tau^2 = (1 - (v_p/c)^2)c^2 dt^2 - \left[\frac{dr^2}{(1 - (v_p/c)^2)} + r^2(d\theta^2 + \sin^2(\theta) d\phi^2\right], \quad (12)$$

where the spatial spherical coordinates are r, θ, ϕ, the t is not transformed. For a very simple spheroid where the matter that produces the gravitational field is distributed in a perfectly homogeneous, spherically symmetric manner that does not varying in t, then (12) is the Schwarzschild solution. In this case, $(v_p/c)^2 = r_s/r$, where $r_s = 2MG/c^2$, M is the mass, and G is assumed to be a universal constant. Until changed, this special case is assumed in what follows. As used in [4,A,B,C], the measures considered in (12) and required to make the left-hand and right-hand sides equal are infinitesimal-light clock and Einstein measures. However, as I have always stressed, these measures are but analogue models for real world measuring devices. In particular, due to the results in [B] relative to subatomic behavior, probably any reliable clock such as an atomic-clock and even a digital watch will suffice. Then we have the notion of Einstein measures needed to measure distance such as r. This is a form of radar measurement and seems to yield a dynamic coordinate system. However, in the actual derivation in [4,C], it is argued that such distance measures, r, for this coordinate system need only be done in the same manner as a standard ruler measurement. Even if the distance measures remained dynamic, the notion of slow transport still yields a fixed unaltered measurement for distance r. This corresponds to the Lawden statement about the Fitzgerald length contraction in that it "can have no physical consequences" [7, p. 12]. Independent from any required measurement schemes, the metric does not exist for points on the Schwarzschild boundary (i.e. the spherical surface where $r = r_s$).

It is claimed that another transformation, the Kruskal-Szekeres transformation (KST) [7, p. 167], eliminates this problem for $r = r_s$ even though the KST does not seem to satisfy the RG requirement of regularity at the points where $r = r_s$. Regularity is an important property in RG that, in general, needs to be satisfied at the coordinate points investigated. I point out, that for the Schwarzschild metric r_s is also called an *event horizon.* There is another surface called the *infinite redshift surface.* For electromagnetic signals, the event horizon is like a "one-way membrane." The infinite redshift surface appears to an observer at a distance $r > r_s$ to shift the photons wave-length λ_r so that $\lim_{r \to r_s +} \lambda_r = +\infty$.

These surfaces need not correspond [3]. For the Schwarzschild solution, these surfaces do correspond.

After taking the KST, when test particle behavior is analyzed, a great deal of new physical behavior is exhibited. This physical behavior is used to strengthen many a science-fiction story. There is yet another transformation that leads to physical behavior that contradicts behavior exhibited by KST and other accepted transformations [3]. But, how is this possible? An explanation for this is related to how one physically interprets the new transformed metric where the interpretation is not related to the rules of RG. GR uses many RG rules but adjoins additional physical interpretations to obtain the GR theory.

For one simple example, Lawden [7,pp. 156-158] transforms the Schwarzschild metric (12) by the Eddington-Finkelstein transformation. In RG, you can consider a geodesic at a regular point in any direction. First, consider the Schwarzschild metric and a freely falling test particle for a fixed $\theta,\ \phi$. The initial conditions state that we have "a particle falling freely along a radius towards the center of attraction in the region" $r > r_s$. "Taking the initial conditions for $t = 0,\ r = R,\ dr/dt = 0$" [7, p. 156], Lawden calculates from the geodesic equations that the equation of motion is

$$\left(\frac{dr}{dt}\right)^2 = r_s c^2 \left(1 - \frac{r_s}{R}\right)^{-1} \left(1 - \frac{r_s}{r}\right)^2 \left(\frac{1}{r} - \frac{1}{R}\right). \qquad (57.2)(13)$$

Notice that the initial conditions are that at $r = R$ the t-clock is set at 0 and the particle's directed speed (velocity) $dr/dt = 0$ and all are measured by a standard coordinate system that is fixed in the gravitational field at R, where the coordinates are all independent from one another. Apparently, the negative root for (13) is taken so that $dr/dt < 0,\ t > 0$. However, I point out that using the RG language these coordinates are really 4-D "surface" coordinates. From this, solving for t, the geodesic equation of motion is

$$ct = \left(\frac{R}{r_s} - 1\right)^{1/2} \int_r^R \frac{x^{3/2}\,dx}{(x - r_s)(R - x)^{1/2}}. \qquad (57.3)(14)$$

The expressions (13) and (14) are not obtained in [7] from (7) but are obtained from an equivalent expression. In this case, the t and the r are still the same measurable notions and one can view the r measures from a spatial rectangular 3-D space with time t as an additional parameter since, technically, r is also a function of t. Since only r and t are related, this behavior can also be "viewed" along r from R. A general experiential description states, "particle is moving towards the center of attraction." Classical GR reduces to solutions for differential equations. These can have unique solutions if one considers specific *initial conditions* like those stated above. But, notice that for the dr/dt and the time coordinate t, the particle has the properties that

$$\lim_{r\to r_s+} \frac{dr}{dt} = \frac{dr}{dt}|_{r=r_s} = 0, \ \lim_{r\to r_s+} t = +\infty. \qquad (15)$$

This is rather unusual behavior. And, there is more. The test particle has a nonzero absolute maximum speed at some r' such that $r_s < r' < R$. Why is this not part of our experience with gravitational fields? The reason is that for such a spheroid of the mass of the Earth, $r_s = 9$mm.

In RG, a path of motion on a surface is determined by an auxiliary parameter $w = t$. Now in GR, if one can view the path of motion from a 5-D space, then 4-D (r, θ, ϕ, t) do not form a mathematically linear path. In order to confuse the matter, one might apply RG geometric terms and state that this gravitational field "curves" or "warps" space-time. But, this geometric language has no actual physical meaning since it can be re-expressed in terms of "forces." Indeed, as properly viewed from local 3-D rectangular space, the path of motion is linear and its physical behavior is determined by the auxiliary 1-D time interval. There is nothing mysterious about any of this when interpreted correctly. Unless one is willing to accept a logical contradiction, other theoretical interpretations for such quantities as the Riemann-Christoffel tensor tend to imply that this interpreted view of particle's behavior does not yield its complete behavior. Here is where the rules for RG and GR differ greatly.

Eddington, and then many years later Finkelstein, suggested a transformation (EFT) for the "time" coordinate. When such transformations are made, care must be taken when one suggests that "time," as we experience it, is being changed. In particular, $u = t + (1/c)(r + r_s \ln |r - r_s|$. With this substitution made, then one gets for Lawden's form of ds^2,

$$ds^2 = r^2(d\theta^2 + \sin^2(\theta)\, d\phi^2) + 2cdr\, du - c^2(1 - r_s/r)du^2. \qquad (57.13)(16)$$

For the same restriction that θ and ϕ are constant, the geodesic satisfies

$$\frac{dr}{du} = r_s c\left(\frac{1}{R} - \frac{1}{r}\right) - c\left(r_s\left(1 - \frac{r_s}{R}\right)\left(\frac{1}{r} - \frac{1}{R}\right)\right)^{1/2}, \qquad (57.18)(16.5)$$

where $dr/du(R) = 0$, $dr/du(r_s) =$ nonzero real number, and for the directed speed (velocity) $\lim_{r\to 0+}(dr/du) = -\infty$ as "viewed" from R. The paths of motion are solutions to differential equations and require initial conditions. In this case, they include that "r must decrease initially" [7, p. 158] meaning that near to but less than R, $dr/du < 0$. This leads to the negative directed speed dr/du. For the identical initial conditions $u = 0$, when $r = R$, solving for u, the u-time will be finite when $r = r_s$.

What is actually obtained physically by this transformation? The g_{ij} that appear in (16) must satisfy the Hilbert-Einstein tensor expression for a gravitational field. But, as described by geodesics and other actual physical behavior, *this represents a new gravitational field.* This is why the theory is called the General Theory of Relativity. The term

General means that there are infinitely many different fields that satisfy the tensor expression. (If an actual gravitational field is not Schwarzschild, then this would imply that the simple non-reversible P-process used in [4] is either more complex or the P-process is further altered when it is realized as part of the natural world.) Lawden analyzes the freely falling particle and uses the *same general and intuitive notions* we associate with space and time measurements and states, "passage through the Schwarzschild radius (surface) being unremarkable" [7, p. 158] for, by (16.5), $\frac{dr}{du}|_{r=r_s}$ is finite and $\neq 0$. However, from the RG viewpoint, the transformation does not have the required regularity at any of the points where $r = r_s$. Since this test particle is really not a mathematical point, then this fact need not be significant.

Although, from RG, if *all of the formulas for geometric measures are transformed* you will get the same numerical measures, this is not what is done for the physical interpretation for the real local Euclidean-like world in which we live. Physically, even though the "times" t and u do not behave in a similar manner, general statements are made relative to the EFT and the behavior being exhibited in terms of our usual intuitive and standard comprehension of time, distance, directed speeds, etc. *Our experiences and intuition have not been transformed since we are making comparative statements.* (The theory of correspondence has not been transformed.) From this view, using a descriptive approach, we do have that the particle crosses the Schwarzschild surface; and, when it passes the surface, it moves faster and faster towards the center of attraction. Also, the infinite redshift surface and event horizon are still both located at $r = r_s$. Notice that equations (14) and (16.5) when solved for u, are actually flight-time expressions that present these forms only because of the selected initial conditions for $t = 0$ and $u = 0$.

These interpretation methods are not the only GR rules that are not part of RG. This is also relative to the 1600s experiment. In that experiment, the scientist had the gravitational field with which to experiment. He lived in it, as do we. The clever inventor made a clock that was distorted so that it gave different results than the observed ones. This is why the scientist knew that the differential clock was faulty. One could do the same thing here and make a distorted u-clock. The problem is that we don't know how the gravitational field behaves near to r_s. We are trying to find the correct field expression so that particles will behave, not just appear to behave, in an "acceptable" manner. This GR rule does not correspond to any of the classical RG rules. *The accepted methods and actual words and thoughts about how moving bodies are to be described have not been transformed.* The behavior is still compared to a local rectangular 3-D space, where the time u varies over an auxiliary 1-D interval.

In order to select what may be the "correct" gravitational field behavior, the *completeness* or *maximal* properly is often added. Essentially, this non-RG rule requires physical entities to appear or disappear only at singularities such as at $r = 0$ in the Schwarzschild case. The KST for the Schwarzschild metric appears to be just such a maximum transformation.

> Somehow, by means of the coordinate transformation that leads to the Kruskal-Szekeres coordinates, one has analytically extended the limited Schwarzschild solution for the metric to cover all (or nearly all) of the manifold [9, p. 833].

But, what is material reality? One might define local material reality relative to a collection of common human senses and comprehension; that is, those defined senses the vast majority use to observe local evidence.

> Though Kruskal's work is undoubtedly of high theoretical interest, does it have any practical application? At present, perhaps not. Kruskal space would have to be *created in toto. . . .* There is no evidence that full Kruskal space exists in nature [11, p. 164].

In [4,B,C], it is shown how all of the time-dilation results follow from comparing behavior with a local standard. From the viewpoint of clocks and the language of GR, one considers freely falling local light-clocks momentarily at rest and orientated along the radius r. The phrase momentarily at rest means that $dr = dR = 0$. But this fact holds if you simply consider r and R as fixed, for $\Delta r = \Delta R = 0$ implies that $dr = dR = 0$. The infinitesimal requirements will shortly be obtained by a different means. *All time-dilation results obtained in* [4] *are obtained by using a fixed method, one part of which is this exact observational and comparison technique.*

There are two approaches to time-dilation concept. One is via gravitational potentials and proper time. The other is found in [4,B,C]. There it is done from the viewpoint of coordinate time, where t^s is coordinate infinitesimal light-clock determined time that is not affected by a gravitational field and, by comparison, t^m is an affected and altered coordinate infinitesimal light-clock as viewed from a nonstandard substratum. Since the entire time-dilation notion in [4,B,C] is based upon rates of change, comparison and not the actual time measures, the alterations in t^m should have an appropriate interpretation. As shown below, the "slowing down" of physical behavior is relative to the digital counts of approximating real light-clocks or equivalent devices like atomic-clocks. The proper interpretation appears on pages 58 and 84 of [4]. The unit of time T is the "tick" that can be emitted when the digital output changes by one digit. If $r_s \leq r \leq R$, r and R are fixed positions and the t^s is eliminated, then the actual time-dilation expression (*) [4, pp. 62] yields

$$\left(1 - \frac{r_s}{r}\right)^{1/2} \int_{t_0^r}^{t_1^r} dt^r = \left(1 - \frac{r_s}{r}\right)^{1/2} \Delta t^r = \left(1 - \frac{r_s}{R}\right)^{1/2} \int_{t_0^R}^{t_1^R} dt^r = \left(1 - \frac{r_s}{R}\right)^{1/2} \Delta t^R.$$

Substituting into this expression, one obtains $(1-r_s/r)^{1/2}\Delta q^r = (1-r_s/R)^{1/2}\Delta p^R$, where $\Delta q^r, \Delta p^R$ are the differences in the observed number of atomic-clock ticks at r and R, respectively. These numbers are independent from the observer. *The result only has meaning in the sense of comparison.* This equation must be properly interpreted. When it is applied to a specific problem, it applies locally to objects at each instant, for the field at r and R, that behave like test particles with respect to the mass M. This interpretation has been experimental verified since it corresponds to the one needed in order to predict the gravitational redshift [4, p. 66]. Let $(1-r_s/r)^{1/2} = 1/4,\ (1-r_s/R)^{1/2} = 1/2$. Suppose that from position R, two almost identical atomic-clocks are constructed and within all forms of observation the clock's tick at the same rate at R (i.e. one tick length with acceptable error is the same as the other tick). One of the clocks is slowly moved to the position r. The field separately affects each clock, which using the language of GR may be considered to be in free fall along the radius but momentary at rest. You observe, from the substratum, the atomic-clocks at R and r. You display the observed t^R counter numbers at the top of a screen and the counter numbers for your atomic-clock t^r at the bottom.

You observe, that the t^R-counter reads $t_0^R = 12955$ ticks at R and this approximately corresponds to your $t_0^r = 2345666$ ticks at r. Then you wait until your t^r-counter number reads $t_1^r = 2345667$ ticks at r and note, immediately, that the t^R-counter number is very nearly $t_1^R = 12957$ ticks at R. From observation, the number of ticks, 1, at r corresponds to the number of ticks, 2, at R. These values satisfy the prediction that $(1/4)\Delta p^r = (1/2)\Delta p^R$. Or, 1 tick at r corresponds to 2 ticks at R. So, arbitrarily using the one r-tick at r as your time unit, you can state that the atomic-clock at R is "speeding-up," at R, at the rate of 2 r-ticks for each 1 r-tick. But, this can also be stated in terms of the R-tick units of time for the atomic-clock at R. In this case, an R observer can state that the atomic-clock at r is "slowing-down" at the rate of 1/2 R-tick for each one R-tick.

However, the time units selected are not relevant for the calculus. Since the calculus is being used, both of these atomic-clocks should be correlated to an ideal "continuum clock" represented by the usual moving hands on a clock-face. *We need to "infinitesimalize" time*, which is formally done by infinitesimal light-clocks. *This allows the atomic-clock digit changes to be measured in terms of a single time unit μ that is not affected by any relativistic alterations as viewed from the substratum or from $\approx +\infty$. This μ can be correlated to the terms sec., min., hrs, etc. which all represent universal units that are not altered by relativistic alterations.* Let sec. mean the unit "a second" for an unaltered clock. Hence, at r, 1 sec. is equivalent to 2 sec.s at R. Suppose that an unaltered light-clock tick takes 2 sec.s, then it might be stated that 2 sec.s at r correspond to 4 sec.s at R. Obviously, this leads to a misunderstanding as to what is actually being altered - the atomic-clocks. (See the Hawking's description [E]).

The results are simply that the behavior of the atomic-clocks is altered by the gravitational field and this alteration is observed by noting the different counter numbers. Taking into account how "light" behaves when moving radially, it seems that these alterations are real and not an illusion. Moreover, there is a deeper physical reason for these alterations.

For the Schwarzschild field and $r_s,\ r_{1_s}$, if energy changes obey the time-dependent Schrödinger equation and GR generally applies, then as shown in [4,B] if ΔE^s is the change without a gravitational field, then at position r the change is $(1 - r_s/r)^{1/2}\Delta E^s = \Delta E^r$. This yields the relation $(1-r_{1_s}/R_1)^{1/2}\Delta E^R = (1-r_s/R)^{1/2}\Delta E^{R_1}$. To measure relativistic alterations in vibrational rates of change via frequencies, experimental evidence requires that you use a fixed universal unit of measure at one observational location [4, p. 66 implies p. 43]. There are additional GR field requirements. The frequency must be associated with an object that behaves like a test particle at $R,\ R_1$ and, as previously, the language of GR might state this as the particle being momentarily at rest at $R,\ R_1$ during radial free fall. Hence, for any electromagnetic radiation produced by such energy changes,

$$\left(\frac{1 - r_{1_s/R_1}}{1 - r_s/R}\right)^{1/2} = \frac{\nu_1^{R_1}}{\nu^R}. \tag{17}$$

Equation (17) predicts gravitational redshifts [4,B,C]. Consider $M_1 > M$. Hence, $r_{1_s} > r_s$ implies that $\nu_1^R < \nu^R$, where the standard for a specific form of radiation is determine for the mass M. For the Sun and the Earth and frequencies measured at the same distance R from the centers of attraction, $\nu_1^R = 0.9999979\nu^R$. There are other derivations for these gravitational effects [7, p. 152 - 153]. But, this one gives an actual physical bases for this behavior rather than some type of general time-dilation. After this derivation was obtained, it was discovered that Einstein conjectured that gravitational redshifts are somehow-or-other associated with the atomic structures. I also point out that the exact same approach yields the Special Theory redshift evidence that is claimed to be but general time-dilation. This behavioral approach is based totally upon comparing what science actually measures - real observed physical behavior. Due to how the line-elements are obtained via electromagnetic propagation theory, one can rationally assume that all alterations in physical behavior due to both the Special and General theories is primarily caused by an electromagnetic interaction which is mediated by the NSP-world entities discussed below.

This appears to be enough information to justify, for the behavior of objects affected by a gravitational field, that RG, with its geometric language, is mainly an analogue model for the behavior of entities that are affected by a gravitational field. What this means is that all such predicted behavior needs to be translated into a meaningful physical language that has meaning for our "real" local Euclidean-like world.

An Expanding Universe Without RG

Considerable effort has been and is being expended in further examining an "expanding universe" in the sense that there is some sort of entity that comprises space that is not merely a coordinate frame. For the basic expansion notion using the RG approach, the device of a 4-D spatial “space” with the time coordinate as a separate parameter is employed [7, pp. 176-179]. Technically, this is a 5-D space; and if you wish to “view” what is going on, you need a 6-D space in which to embed it. There is also the *Copernicus* or *Cosmological principle.* This is known to be a philosophic idea that is forced upon the model. Indeed, using the 4-D spatial approach the Robinson-Walker metric is obtained after some clever substitutions [7, p. 177- 181]. By applying this metric at every location, one can “see” aspects of the Copernicus principle with the famous balloon analogy. Pennies pasted on the surface of a rubber balloon will all appear to recede, one from another, as the balloon is inflated. Of course, one could also make little black ink-marks on the balloon, and then the ink-marks will not only seem to recede from the other ink-marks but probably the ink-marks will also expand. Anyway, the balloon's rubber material is supposed to represent the unknown entities of which the universe is composed.

There is only one problem with the RG approach. It is often not necessary. As shown in [4], using the exact same method that predicts, without RG, metrics such as the Schwarzschild metric, linear effect line-element and an equivalent Robinson-Walker metric are derived. This derivation method is not varied and uses simple physical-like behavior as viewed from the predicated “subparticle field” as discussed below. The derivation yields an apparent radial expansion of a realized universe from either a single location, a collection of locations or, for those that simply must accept the not verified Copernicus principle, for every standard location within our universe. This <u>expansion</u> is isotropic when “viewed” from each point of application, but need not be related to a homogeneous matter density. The metric obtained in [4] is

$$dS_e^2 = (cdt^s)^2 - \frac{(dR^s)^2}{1-(R^s)^2/(ca)^2} - (R^s)^2(\sin^2\theta^s(d\phi^s)^2 + (d\phi^s)^2) \qquad (18),$$

where equation (18) is (29) in [4, p. 68]. In the derivation [4, p. 60], we establish that $\theta^m = \theta^s$, $\phi^m = \phi^s$, $R^m = R^s$, where the superscript s indicates that no expansion compensating alterations in local measurements are included.

This metric relates a subparticle component potential velocity R^s/a to the standard world without any gravitational field present. (For comprehension, you can view this as a type of subparticle (substratum) expansion.) This corresponds to the Robinson-Walker (RW) metric (60.13) as it appears in Lawden [7, p. 179], where Lawden's $\sigma = R^s/S$ and $a = S/c$. After introducing a special smoothed-out form of matter for homogeneity, applying (18) at every location and the Hilbert-Einstein equations, this yields the closed Friedmann model. If R^s/a is zero, then you get Euclidean space. Now many are not aware of the fact that if one just does not accept the metric with the negative term such as in (2) but rather requires that this comes from a coordinate system applied to a Euclidean-like process, then the time coordinate would actually be a pure complex number ti [7, p. 8]. Coordinate systems that use complex numbers are used throughout physical science to model a great deal of behavior. With this in mind, if you replace R^s/a with $(R^s/a)i$, then the metric you get is for the open Friedmann model. One the other hand, if one is not concerned with "units" being conserved, then an RW type metric can be obtained for an open Friedmann model by a coordinate transformation [7, p. 197]. If you had a photon emitted from a location at a distance $R^s \neq 0$ from your origin $R^s = 0$, then using photon wave properties the exact same derivation used to generated the cosmic redshift for the RW metric [7, p. 182] yields the same redshift evidence for (18). Note that you can do various things with R^s/a, by ad hoc choice, at any moment in the expansion. For example, make it $dR^s/dt^s \neq 0$ or R^s/a a constant. But, I wrote the phrase "subparticle field" and this expansion is relative to such a "field." What is a subparticle field?

The mathematical structure constructed in [5] is the only one of which I am aware that can actually write-out, on a computer-like screen, part of its own theory predictions. Photons, although they seem to interact via quanta of energy, also appear to have a continuous energy spectrum. This further corresponds to particles that are said to be *free in space.* Even if this spectrum is slightly granulated, the set of statements {A photon with energy measured as 1/n}, where n is considered as varying over the set of natural numbers, characterizes actual different photon behavior within our universe. If you encode these statements and embed them into the mathematical structure constructed in [5], them Theorem 9.3.1 [5, p. 83] rationally predicts the existence of an entity that is characterized as actually having an infinitesimal amount of energy. How it predicts this is that the resulting statements are decode and many of them read "A photon with energy measure 1/ ." but where the symbol "n" should be is now a blank space. The reason it is blank is because there is no symbol in the set of symbols used to represent the natural numbers that fits into the blank space. So, we use a new symbol, say ϵ, and insert it since we do know how the ϵ behaves. This infinitesimal quantity ϵ is an actual "number" with basic properties described within Nonstandard Analysis. Since first suggested in the late 1600s, these are the numbers it took 300 years to discovery mathematically.

Although diagrams may aid in comprehending particle behavior within the subatomic regions, the facts are that such particles are only differentiated one from another by their described characteristics. Subparticles take this notion to the most extreme in that they are only operational defined via characterizing components. I will not discuss them in this article in any detail since their properties are formally discussed throughout many articles, monographs [3,5] and books. But, as an example, certain subparticles are taken as objects that model infinitesimal statements like $(x + dx, y + dy, x + dz)$. Relative to a coordinate system within the nonstandard physical world (NSP-world), these coordinates can be perfectly represented by infinitesimal light-clock counts and there is a subparticle that has these representative position coordinates. When the significant *standard part operator* is applied to this subparticle, the result has the position coordinate measure of (x, y, z). This is considered as the position coordinate of an entity within the material world in which we dwell.

Using this approach but in a somewhat more complex manner, every entity within our universe can be produced by combinations of but one type of subparticle termed the *ultimate subparticle.* To apply this to (18), one simply considers a potential velocity subparticle component associated with the measure R^s/a. All material entities would require this additional subparticle effect when they are realized as actual physical entities via the standard part operator. All entities that exist within our universe appear to recede from the NSP-world positions of application. The effect is most easily understood if one considers a universe that expands from but one or a small collection of NSP-world locations.

Subparticles are considered as physical-like entities that correspond directly to the infinitesimal part of the infinitesimal calculus. Should we take their existence seriously? Many years ago, there was a discussion about certain quantum quantities that are not preserved during certain physical scenarios. There were two choices. Either accept that, *for specific scenarios*, these quantities are not preserved or to postulate the existence of the neutrino that would acquire the "missing" quantum data. The physics community decided to accept the neutrino hypothesis. But, the mathematics of quantum theory did not postulate the neutrino. It was human intervention that rejected the scenario related non-conservation notion and accepted the neutrino. In the case of subparticles, based upon directly observed evidence, the mathematical structure automatically contains the basic information about these objects and the only human intervention is to locate statements that describe their behavior. These assumed objects do seem to "explain," in a rather simple and even better manner, much physical behavior. Indeed, the major mathematical tool used by science, engineering, etc. is the infinitesimal calculus. This is extraordinary evidence for the acceptance of subparticles. There is the question as to how the R^s, which is a measure of distance from the position(s) where the expansion "starts," is controlled within the NSP-world. There is a reason for excluding any detailed speculations. This is what Theorem 7.3.4. [5, p. 68] implies. There must always exist within the NSP-world for various developing universes, the *ultranatural events*. We slightly know, in general, how these ultranatural events behavior relative to each other and natural events; but, using any language developed within our universe, we can not describe in detail what each event comprises.

References

1. Eisenhart, L. P., An Introduction to Differential Geometry: with the use of tensor calculus, Princeton University Press, Princeton, NJ, (1947).

2. Herrmann, R. A., Encoding of Quantum State Information Within Subparticles (1999) http://www.arxiv.org/abs/quant-ph/9903038

3. Herrmann, R. A., General Relativity and Contrary Predictions, (1999) http://www.arxiv.org/abs/9905040

4. Herrmann, R. A., Nonstandard Analysis Applied to Special and General Relativity - The Theory of Infinitesimal Light-Clocks, (1994). The most recent version http://www.serve.com/herrmann/cont4.htm or an older version http://www.arxiv.org/abs/math/0312189

5. Herrmann., The Theory of Ultralogics (1979 -). The most recent version http://www.serve.com/herrmann/cont3.htm or an older version http://www.arxiv.org/abs/math/9903081 and http://www.arxiv.org/abs/math/9903082

6. Herrmann, R. A., Mathematical philosophy and developmental processes, Nature and System 5(1/2)(1983):17-36.

7. Lawden, D. F., An Introduction to Tensor Calculus, Relativity and Cosmology, John Wiley & Sons, New York, (1982).

8. Mehra, J., Einstein, Hilbert, and The Theory of Gravitation, D. Riedel Publishing Co, Boston, (1974).

9. Misner, C. w., K. S. Thorne and K. A. Wheeler, Gravitation, Freemann, San Fran., (1973).

10. Patton and Wheeler, "Is physics legislated by a cosmogony? In Quantum Gravity (Ed) Isham, Penrose, Sciama, Oxford University Press, (1975):538-605.

11. Rindler, W., Essential Relativity, Springer-Verlag, New York, (1977).

Additional References

A. Herrmann, R. A., "A hypercontinuous hypersmooth Scwarzschild line-element transformation," Internat. J. Math. and Math. Sci., 20(1)(1997):201-204.

B. Herrmann, R. A., "An operator equation and relativistic alterations in the time for radioactive decay,"," Internat. J. Math. and Math. Sci., 19(2)(1996):397-402.

C. Herrmann, R. A., "Operator equations, separation of variables and relativistic alterations," Internat. J. Math. and Math. Sci., 18(1)(1995):59-62.

D. Herrmann, R. A.. "Consecutive points and nonstandard analysis." Math. Japonica, 36(1991):317-322.

E. This unit notion can be stated using the S. Hawking statement on page 25 of "A Brief History of Time, Bantam Books, NY, (1988). "The astronomer sees the watch reach 11:57 a.m. After an hour the watch reaches 11:58 a.m." The clocks used here represent ideal continuum clocks and the unit of time for both is the universal minute. The observed r clock changes by one min., while the R clock changes by 60 mins.

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